Lecture I (1) Present the scheme of argument-

(2) Then discuss recommended books.

(3) Discuss philosophical relevance of E.P. Phys
& explain philosophical topics
a.) Theory change
2) theory– expt
c.) Atomistic programme

(4) Present Table of the particles
+ classification of the interactions

(5) Explain range-energy formula

(6) Explain chronological Table up to 1940
stressing significance of discovery
of the positron and the meson.

Scheme of argument

Philosophical Topics {
Intertheory relationships
Relation of theory to experiment
Atomistic programme

↓

History 1897–1973

↓

3 KEY CONCEPTS → Answers to some philosophical questions

(1927) 1. The Quantized field
(1949) 2. The Feynman diagram
↳ Renormalization → Lamb shift
(1958) 3. The analytic S-Matrix
↳ The Bootstrap

amount published since 1930
on elem. particles ~ amount
published up to 1930 all
branches of physics. (encyclop)

EPT & Phil Sci   Summary

① What is relevance of EPT to Phil. Sci.?

- a.) an up to date case study for ideas about progress in Science
- b.) Foundation for the reductionist programme

② Philosophical Topics

- a.) Inter theory relationship
- b.) Relation of theory to experiment
  - 1) Predictive power
  - 2) Computation gap
  - 3) Crucial experiments
- c.) the Atomistic Programme.

③ Topics not discussed

- a.) Symmetry
- b.) Theory of Measurement in rel. field theory etc
- c.) Ontological status of the particles

④ Table of the particles

⑤ Chronological Table

Notes: apparent predictions, positron & meson
True predictions, anti-proton, K° regeneration, $\Omega^{\cancel{+}}$, neutrino.
crucial expts Lamb shift & Ω^-

⑥ Key Concepts

(1927) a) the Quantized field
(1949) b) Feynman diagrams: → renormalization
↳ Lamb shift
(1958) c) Analytic S-matrix
↳ Bootstrap hypothesis

⑦ Philosophical Conclusions

a.) Field theory → S-matrix theory constitute a paradigm shift ([illegible] or directional shift within a paradigm)

b.) In both cases calculation gap has largely led to a degeneration of the research programme. Early successes have tended to die out – Feynman diagrams gave a great computational boost to field theory but did not constitute a paradigm shift.

e) Theories develop by

1) simple modification (Klein Gordon Eq. ~~Strangeness & NU~~)
2) analogical extension (SU2 → SU3)
3) Reformulation method in which old theory is reformulated, then is used as basis for extending the theory in some new direction.

Lecture II

1.) Explain

hadrons
mesons   Baryons.

→ simple picture of nucleon surrounded by meson fields

weak interactions to be understood in terms of interactions of 2 currents

$$H' \sim \int J_\mu(x) J_\mu^\dagger(x) \, d^3x .$$

where $J = J_L + J_B$

So H' involves.

$J_L^2 \rightarrow$ leptonic decays $\mu^{(-)} \rightarrow e^{(-)} + \bar{\nu}_e + \nu_\mu$. ~~$n \rightarrow p + e^{(-)} + \bar{\nu}_e$~~

$J_L J_B \longrightarrow$ semi leptonic. $n \rightarrow p + e^{(-)} + \bar{\nu}_e$

$J_B^2 \rightarrow$ non-leptonic decays. $\Lambda^0 \rightarrow \underset{(p)}{N} + \underbrace{\underset{(n)}{N} + \underset{(\bar{p})}{\bar{N}}}_{\pi^-}$

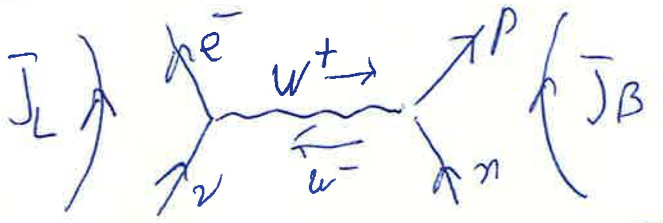
Note 2 stages may be involved as also in

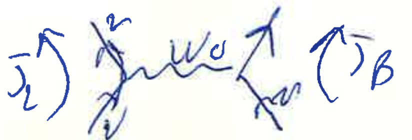
$$\pi \longrightarrow N + \bar{N} \longrightarrow \mu + \bar{\nu}_\mu \text{ or } \bar{\mu} + \nu_\mu .$$

note $\pi^0 \rightarrow 2\gamma$

$$(\pi^+ \rightarrow p + \bar{n} \rightarrow \mu^+ + \nu_\mu)$$

note charged current interaction appears to operate



Can we observe 

i.e. neutral current interactions as in e.m. interactions

this has led to Weinberg's adoption of a unified theory of e.m. & weak interactions.

⇒ notice difference between $K_L = K_0 + \overline{K_0}$ not an eigenstate of S
and $|K_0\rangle \otimes |\overline{K_0}\rangle$ which is an eigenstate of
$S (= S_1 + S_2)$ with eigenvalue 0
and so could annihilate
→ 2π's say by strong
interaction

2) Discuss discovery of the stable hadrons:
pions
V-particles → hyperons.
Kaons.

artificial production 1948 synchrocyclotron
1953 Cosmotron at Brookhaven.

→ Pais theory associated production (1952)

Ex $\pi^+ + n \to \Lambda^0 + K^+$

assignment of strangeness Q. No by Gell-Mann & Nishijima 1953—1955.

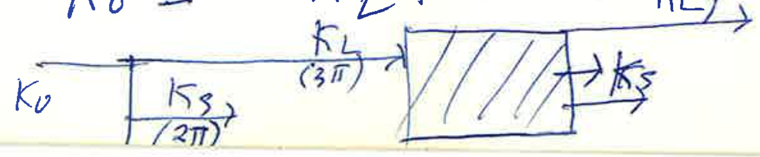
K₀ regeneration K^0 produced in reaction

$\pi^- + p = \Lambda^0 + K^0$

then $K^0 \to \pi^+ + \pi^- \to \overline{K_0}$ by weak interactions
$\pi^0 + \pi^0 + \pi^0$

Hence K^0 gradually turns into a $\overline{K_0}$ — not possible for hyperons by conservation of baryon no — not possible for π^0, η^0 since they are identical with their antiparticles.

$K_0 = K_L + K_S$ $\overline{K_0} = K_L - K_S$
also. $K_L \sim K_0 + \overline{K_0}$



3.) Discovery of the anti-proton using the bevatron.

Now discuss quantized field (Lecture III)

→ Feynman diagrams → renormalization:
and Lamb shift
Lecture IV

→ approximation schemes in hadron physics ⇓
Tamm - Dancoff
Tomonaga.
↓
Chew-Low-Wick theories
↓
analytic S-matrix
, the bootstrap.

Lecture V The analytic S-matrix & the Bootstrap

Lecture VI Philosophical conclusions.

Quantized Field Theory   Lecture III

3 points of view

1) a reformulation of N-particle S Eq — 2nd quantization

2.) Quantization of an observable field

3.) an autonomous discipline
with new axioms in the form
of commutation properties of the
field variables

books Schweber, Bethe, de Hoffmann:
Mesons, Fields vol I Fields
Wentzel : Quantum Theory of Fields
(3rd ed.) Heitler : The " " " Radiation
Jauch, Rohrlich : The theory of Photons, electrons

The Quantized Field

KEY CONCEPTS ⑧

Consider sequence:—

~~N particle S. Eq~~

① N classical particles $\xrightarrow{\text{1st quantization}}$ N-partial Sch. Eq. $\underset{\text{equiv}}{\overset{\text{2nd quantization}}{\Rrightarrow}}$ quantized field

or
② classical field $\xrightarrow{\text{Field quantization}}$ quantized field $\Rrightarrow$ N-particle S. Eqn.

② corresponds to treating continuous field as a quantum system by quantizing the amplitude of the field at all points x. The Hamiltonian is shown to be the same as for a set of harmonic oscillators, the quantum no (excitation) of each oscillator is then interpreted as the no of particles associated with the mode in question (mode determined by $\underset{\sim}{k}$-value → momentum of the particle. (Fourier component of the field)

We can express ① as follows:— In Hilbert space with symmetrized vectors for N-particles.

$\boxed{0}$ $\boxed{1}$ $\boxed{2}$ - - $\boxed{N}$.

We are concerned in H_N with operators which change vectors within the Hilbert space H_N.

Ed quantized field

scalar field:

$(\Box^2 - \mu^2)\psi = 0 \qquad \Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

$L = \int \mathcal{L}\, d^3x$

~~The $L = -\frac{1}{2}\left\{\sum\left(\frac{\partial\psi}{\partial x_\nu}\right)^2 + \mu^2\psi^2\right\}$~~

$\mathcal{L}$ is lagrangian density

~~$\Pi = \frac{\partial L}{\partial\dot\psi} = \dot\psi$~~

$$L = \frac{1}{2}\left\{\dot\psi^2 - (\nabla\psi)^2 - \mu^2\psi^2\right\}$$

$\Pi = \dot\psi$

Eq. of motion $\quad \frac{\partial L}{\partial\psi} - \frac{\partial}{\partial x_k}\frac{\partial L}{\partial\left(\frac{\partial\psi}{\partial x_k}\right)} - \frac{\partial}{\partial t}\frac{\partial L}{\partial\dot\psi} = 0.$

Energy density

$H = \Pi\dot\psi - L = \frac{1}{2}\left\{\Pi^2 + (\nabla\psi)^2 + \mu^2\psi^2\right\} \geq 0$

Expand. $\psi = V^{-1/2}\sum q_k e^{ik\cdot x} \qquad \Pi = V^{-\frac{1}{2}}\sum p_k e^{-ik\cdot x}$

$\omega_k = c\sqrt{k^2 + \mu^2}$

$q_k = q_{-k}^{*}$

Total energy $\mathcal{H} = \frac{1}{2}\sum\left(p_k^{*} p_k + \omega_k^2 q_k^{*} q_k\right)$

$q_k = \sqrt{\frac{\hbar}{2\omega_k}}\left(a_k + a_{-k}^{+}\right) \qquad p_k = \sqrt{\frac{\hbar\omega_k}{2}}\, i\left(a_k^{+} - a_{-k}\right)$

$N = a_k^{+} a_k$

so $[\psi, \Pi] = i\hbar\,\delta^3(x - x') \rightarrow \quad [a_k\, a_k^{+}] = 1$

$\mathcal{H} = \frac{\hbar}{2}\sum_k \omega_k\left(a_k^{+} a_k + a_k a_k^{+}\right] \rightarrow \frac{\hbar}{2}\sum \omega_k\,(2N_k + 1) \ldots$

$G = \sum N_k\, \hbar k.$

[essentially For the harmonic oscillator $H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2)$

$a = \frac{1}{(2m\hbar\omega)^{\frac{1}{2}}}(p - i m\omega q), \quad a^{+} = \frac{1}{(2m\hbar\omega)^{1/2}}(p + i m\omega q)$

$[q, p] = i\hbar, \quad [a\, a^{+}] = 1$

$H \rightarrow \hbar\omega\, a^{+}a + \frac{1}{2}\hbar\omega \ldots$]

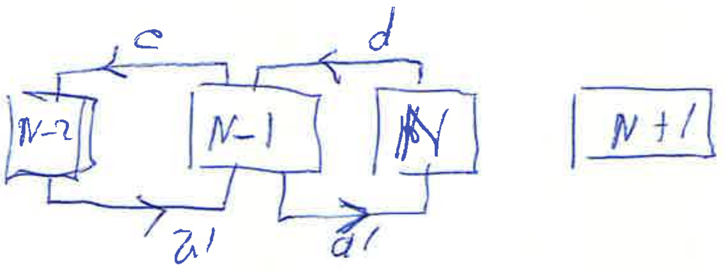
Now introduce Fock space $\mathcal{H} = H_0 \oplus H_1 \oplus - - -$

then consider operators a b c - - which shift vector down one place.

& operators a' b' c' . - which shift vector up one place.

If we consider an operator $F = a'b'cd$ say with equal no. primed & unprimed terms

clearly ~~$F\ \mathcal{H}_N \rightarrow \mathcal{H}_N$~~ $F h_N \in \mathcal{H}_N$ | h_N is vector in $\mathcal{H}_N$



Now we can generalize to consider theories involving operators which involve ~~add~~ unequal no's of primed & unprimed terms

Ex operator a is a destruction operator

a' . . . creation ..

Thus the so-called quantized fields are operators which are built up by superimposing terms linear in destruction and/or creation operators. They enable us to describe

Ex ~~$H = \sum \eta_a^{+} \eta_b ($~~

$$H = \sum_n H_n = \sum_{ab} \eta_a^{+} \langle a|H|b\rangle \eta_b$$

$$= \int d^3x\, \psi^{+}(x)\, H(x)\, \psi(x)$$

where $\psi(x) = \sum a_i u_i(x)$

$\psi^{+}(x) = \sum a_i^{+} u_i^{*}(x)$

$$\langle a|H|b\rangle = \int u_a^{*}(x)\, H\, u_b(x)\, d^3x.$$

If u_i is eigenstate of H

we get $H = \sum \eta_i^{+} \eta_i E_i = \sum N_i E_i$

In quantum field theory of S. Eq. (cf Schiff p 336)

The $L = i\hbar \psi^{*} \dot\psi - \frac{\hbar^2}{2m} \nabla\psi^{*} \cdot \nabla\psi - V(x)\psi^{*}\psi$

Eq. of motion is $i\hbar \dot\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$, $\Pi = i\hbar \psi^{*}$

$\psi = \sum a_k(t) u_k(x)$

$\psi^{*} = \sum a_k^{+} u_k^{*}(x)$ gives $[a_k, a_l^{+}] = \delta_{kl}$ $\left[\psi(x)\Pi(x') = i\hbar\, \delta^3(x-x')\right]$

and $H = \Pi\dot\psi - L = -\frac{i\hbar}{2m} \nabla\Pi \cdot \nabla\psi - \frac{i}{\hbar} V \Pi\psi$

$H = \sum a_k^{*} a_k E_k$ as above

if u's are eigenstates of H

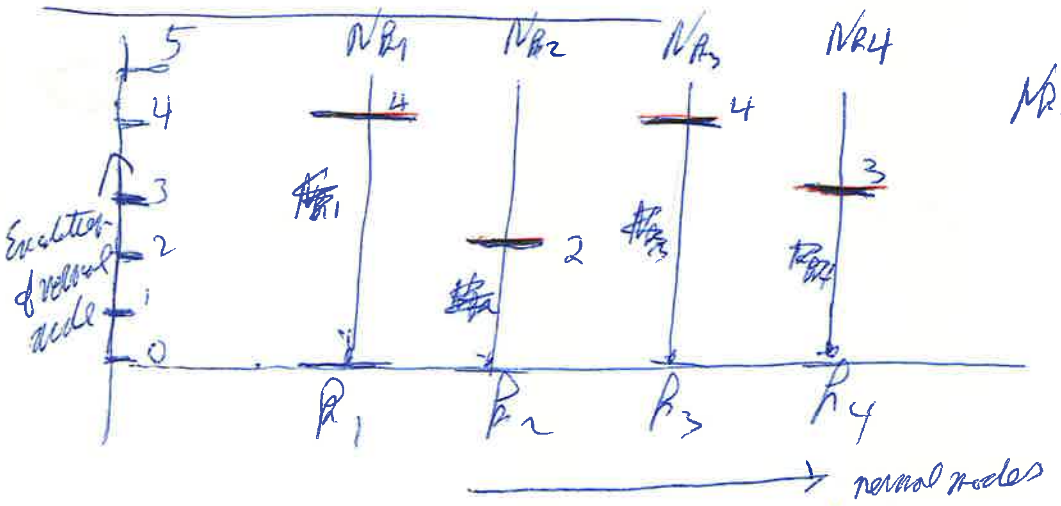
$= \sum N_k E_k$, $N_k = a_k^{*} a_k$.

(notice no zero point energy)

theoretically the creation & destruction of particles.

Here by reformulating the theory with N-fixed, a natural generalization emerges which can describe variable number of particles

Alternative view is this:


5
4
3
2
1
0
Excitation of normal mode
N_{k_1}
N_{k_2}
N_{k_3}
N_{k_4}
4
2
4
3
~~N_{k_1}~~
~~N_{k_2}~~
~~N_{k_3}~~
~~N_{k_4}~~
k_1
k_2
k_3
k_4
normal modes

$N_{k_1} + N_{k_2} \cdots = N$

k's are wave-vectors $= \frac{2\pi}{\lambda}\hat{\lambda}$.

momentum of particle with wave number k is $\hbar k = h/\lambda$, the de Broglie wavelength.

Now move lines so that total N varies instead of being fixed

(& in terms of of frequency, $\omega_1 \; \omega_2 \cdots \omega_3$

Excitation of n° of mode $\omega_i \equiv$ n° of particles with energy $\hbar\omega_i (= h\nu_i)$)

(in Acoustics k is called the wavelength constant
ω is called the "pulsatance")
or "angular frequency"

For charged field note

that $H = \sum (p_k^* p_k + \omega_k^2 q_k^* q_k)$

$e = -i\varepsilon \sum_k (p_k q_k - p_k^* q_k^*)$

write $q_k = \sqrt{\frac{\hbar}{2\omega_k}}\,(a_k + b_k^\dagger)$

$p_k = \sqrt{\frac{\hbar\omega_k}{2}}\; i\,(a_k^\dagger - b_k)$

then $H = \frac{\hbar}{2} \sum \{\omega_k (2 N_k^+ + 1) + (2 \overline{N}_k + 1)\}$

$e = \frac{\hbar\varepsilon}{2} \sum (N_k^+ - \overline{N}_k)$

This brings in the idea of the anti-particle which is due essentially to the fact that ρ is not +ve definite,

$$\rho = \frac{i\hbar}{2mc^2}\left(\psi^* \frac{\partial\psi}{\partial t} - \psi \frac{\partial\psi^*}{\partial t}\right)$$

with $\Phi = \phi(x)\, e^{-i/\hbar\, E t}$

$\rho = \frac{E}{mc^2}\,\phi^*\phi \;\to\; \pm\, \phi^*\phi$ according as $E = \pm mc^2$

This is really due to relativistic possibilities

Note Essentially $H = \frac{E^2}{mc^2}$ is +ve definite

2nd. Quantization

$|X\rangle = |\alpha_1^1\rangle|\alpha_2^2\rangle \cdots |\alpha_N^N\rangle$

$P_r|X\rangle = |\alpha_{r_1}^1\rangle|\alpha_{r_2}^2\rangle \cdots$

$\overline{P_r}|X\rangle = |\alpha_1^{r_1}\rangle|\alpha_2^{r_2}\rangle \cdots$

$[P_r, \overline{P_s}] = 0$ and $\overline{P_r}|X\rangle = P_r^{-1}|X\rangle$

But N.B. $\overline{P_r} P_s|X\rangle \neq P_r^{-1} P_s|X\rangle$

But $= P_s \overline{P_r}|X\rangle = P_s P_r^{-1}|X\rangle$

Consider $S|X\rangle = \sum_r P_r|X\rangle \cdot \frac{1}{\sqrt{n_1! n_2! \cdots \times \sqrt{N!}}}$

(cf alternative normalization if Σ runs over distinct states — the normalization is $\frac{\sqrt{n_1! n_2! \cdots}}{\sqrt{N!}}$)

n_1 is no. of particles in state U^1
n_2 — — — — U^2

where $|U^1\rangle, |U^2\rangle$ are all distinct

$H = H_1 + H_2 + H_3 \cdots H_N$. ~~if $H_r|\alpha_r^n\rangle = E^n|\alpha_r^n\rangle$~~ $H_r|\alpha_r^n\rangle = E^n|\alpha_r^n\rangle$

then $H(S|X\rangle) = \left(\sum_n n_n E^n\right)(S|X\rangle)$

where E^n is energy of state $|U^n\rangle$

So $E = \sum n_n E^n$.

cp. assembly of oscillators

$$E = \sum_r \left(n_r + \tfrac{1}{2}\right) \hbar \omega_r .$$

Identify n_r with excitation of rth oscillator and $E^r = \hbar\omega_r$, and neglect zero-pt energy $\sum_r \frac{1}{2}\hbar\omega_r = \infty$.

Theory harmonic oscillator

$$H = \frac{1}{2m}\left(p^2 + m^2\omega^2 q^2\right)$$

ω is angular frequency.

$$\{q, p\} = i\hbar$$

write $a = (2m\hbar\omega)^{-\frac{1}{2}}\left(p \cancel{+} i m\omega q\right)$

$$a^+ = (2m\hbar\omega)^{-\frac{1}{2}}\left(p + i m\omega q\right)$$

$$[a, a^+] = 1 , \qquad \left.\begin{aligned} p &= \sqrt{\tfrac{m\hbar\omega}{2}}\,(a + a^+) \\ q &= i\sqrt{\tfrac{\hbar}{2m\omega}}\,(a - a^+) \end{aligned}\right\}$$

$$H = \tfrac{1}{2}\hbar\omega\left(a a^+ + a^+ a\right) = \left(N + \tfrac{1}{2}\right) h\omega .$$

$$\underline{N = a^+ a}$$

Eigenvalues of N

Consider $N|n\rangle = n|n\rangle$

$$N(a|n\rangle) = a^{+}a \cdot a|n\rangle$$
$$= (aa^{+}-1)\,a|n\rangle$$
$$= (n-1)\,a|n\rangle.$$

So $a|n\rangle = c_n|n-1\rangle$.

But N cannot have a negative eigenvalue since. $\langle n|N|n\rangle = \||a|n\rangle\||^2 \geq 0$

and $c_n^2 = \langle n|a^{+}a|n\rangle = n$

so $c_n = \sqrt{n}$.

So $a|n\rangle = \sqrt{n}\,|n-1\rangle$

But if n is not an integer this procedure would lead to an eigenstate with a negative eigenvalue. Hence n is an integer and $a^n|n\rangle = |0\rangle$, the lowest state

Similarly $N(a^{+}|n\rangle) = a^{+}a \cdot a^{+}|n\rangle$
$$= (n+1)(a^{+}|n\rangle)$$

$\therefore a^{+}|n\rangle = c_n'|n+1\rangle$ $\quad c_n'^2 = n+1$ so $c_n' = \sqrt{n+1}$

$\therefore a^{+}|n\rangle = \sqrt{n+1}\,|n\rangle$

$$H \cancel{=} H_0 = \sum a_n^\dagger a_n \hbar\omega_n$$

$$= \sum_n a_n^\dagger a_n E^n$$

$$= \cancel{\sum_n} \sum_i \sum_j a_i^\dagger a_j E^j \delta_{ij}$$

$$= \sum_i \sum_j a_i^\dagger a_j E^j \int u_i^*(\mathbf{r})\, u_j(\mathbf{r})\, d^3 r$$

$$\cancel{= \sum_i \sum_j} = \underline{\sum_i \sum_j a_i^\dagger a_j H_{ij}}$$

$$\underline{\text{Or}} \quad = \underline{\int d^3 r \left(\psi^\dagger(\mathbf{r})\, H\, \psi(\mathbf{r}) \right)}$$

$$\psi(r) = \sum a_i u_i(\mathbf{r})$$

$$\psi^\dagger(r) = \sum a_i^\dagger u_i^*(r)$$

and $H u_j(\mathbf{r}) = E^j u_j(\mathbf{r})$

$\downarrow$ 1-particle Hamiltonian.

$\psi(r)$ and $\psi^\dagger(r)$ are 2nd quantized wave functions [satisfying $[\psi(\mathbf{r}), \psi^\dagger(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}')$)

$$\sum_{i>j} V_{ij} = \frac{1}{2} \iint d^3 x\, d^3 y\ \psi^\dagger(x)\, \psi^\dagger(y)\, V(x,y)\, \psi(x)\, \psi(y)$$

$$\left(= \frac{1}{2} \sum_i \sum_j \sum_n \sum_s a_i^\dagger a_j^\dagger V_{ij;ns} a_n a_s \right.$$

$$\left. V_{ij;ns} = \iint u_i^*(x)\, u_j^*(y)\, V(x,y)\, u_n(x)\, u_s(y)\, dx\, dy \right)$$

From $H = \int \psi^+(\underset{\sim}{r}') H(\underset{\sim}{r}') \psi(\underset{\sim}{r}') \, d^3r'$

we recover Eq. of motion $i\hbar \frac{\partial \psi}{\partial t} = H \psi(r)$

from $i\hbar \frac{\partial \psi}{\partial t} = [\psi, H]$

provided ⊛ $[\Psi(\underset{\sim}{r}), \Psi^+(\underset{\sim}{r}')\Psi(\underset{\sim}{r}')] = \delta(\underset{\sim}{r}-\underset{\sim}{r}')\Psi(\underset{\sim}{r})$

since this implies

$$[\Psi(\underset{\sim}{r}), \Psi^+(\underset{\sim}{r}') H(\underset{\sim}{r}') \Psi(\underset{\sim}{r}')] = \delta(\underset{\sim}{r}-\underset{\sim}{r}') H(\underset{\sim}{r}') \Psi(\underset{\sim}{r}')$$

But ⊛ is satisfied for either

$$\left.\begin{aligned} [\Psi(\underset{\sim}{r}), \Psi^+(\underset{\sim}{r}')] &= \delta(\underset{\sim}{r}-\underset{\sim}{r}') \\ [\Psi(\underset{\sim}{r}), \Psi(\underset{\sim}{r}')] &= 0 \end{aligned}\right\} \ \underline{\text{Bosons}}$$

or

$$\left.\begin{aligned} \{\Psi(\underset{\sim}{r}), \Psi^+(\underset{\sim}{r}')\} &= \delta(\underset{\sim}{r}-\underset{\sim}{r}') \\ \{\Psi(\underset{\sim}{r}), \Psi(\underset{\sim}{r}')\} &= 0 \end{aligned}\right\} \ \text{Fermions}$$

⊛ is a rudimentary form of Green's trilinear commutation relations → parastatistics

__Dirac Eq__ $H^2 = p^2c^2 + m^2c^4$ $H \sim mc^2 + \frac{p^2}{2m} + -$ $H = i\hbar\frac{\partial}{\partial t}$

$\underset{\sim}{p} = -i\hbar\underset{\sim}{\nabla}$

$\therefore\ H = \underset{\sim}{\alpha}\cdot\underset{\sim}{p}c + \beta mc^2$

or $i\hbar\frac{\partial\psi}{\partial t} = \left(\left(-i\hbar c\underset{\sim}{\alpha}\cdot\underset{\sim}{\nabla}\right) + \beta mc^2\right)\psi$

now $\dot{x} = \frac{1}{i\hbar}[x, H] = \alpha_x c$

But $\alpha_x^2 = 1$, so eigenvalues are given by

$\alpha_x|\phi\rangle = \lambda|\phi\rangle \ \therefore\ \alpha_x^2|\phi\rangle = \lambda\,\alpha_x|\phi\rangle$

$= |\phi\rangle = \lambda^2|\phi\rangle$

$\therefore\ \lambda^2 = 1$ or $\lambda = \pm 1$

Hence eigenvalues of $\dot{x}$ are $\pm c$

Schrödinger's Zitterbewegung.

Foldy-Wouthuysen transformation gives $\dot{X} = p/m$ as we should expect (cf also Newton-Wigner)

eigenvalues of X are spread out $\sim \hbar/mc$ in x-space

Similar remarks apply to the __Klein-Gordon equation__

$$-\hbar^2\frac{\partial^2}{\partial t^2}\psi = \left(-\hbar^2c^2\nabla^2 + m^2c^4\right)\psi$$

or $\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \left(\frac{mc}{\hbar}\right)^2\psi = 0$

Eigenstates of $N_k = a_k^\dagger a_k$

Bose statistics $\quad a_k a_k^\dagger - a_k^\dagger a_k = 1 \quad$ ①

Fermi " $\quad a_k a_k^\dagger + a_k^\dagger a_k = 1 \quad$ ②

For Bosons
from ① consider Eigenket of N_k with eigenvalue $0 = |0\rangle$

then $N_k a_k^\dagger |0\rangle = a_k^\dagger a_k a_k^\dagger |0\rangle = \cancel{a^\dagger a_k}\, a_k^\dagger (1+N_k)|0\rangle$
$= a_k^\dagger |0\rangle$

So $a_k^\dagger |0\rangle$ is eigenstate of N_k with eigenvalue 1.

For Fermions note
$$N_k^2 = a_k^\dagger a_k a_k^\dagger a_k$$
$$= a_k^\dagger (1 - a_k^\dagger a_k) a_k$$
$$= a_k^\dagger a_k = N_k.$$

where eigenvalues are
$$N_k \phi = \lambda \phi$$
$$N_k^2 \phi = \lambda^2 \phi = N_k \phi = \lambda \phi$$
$$\therefore \lambda^2 = \lambda \text{ or } \lambda(\lambda-1) = 0, \quad \lambda = 0 \text{ or } 1$$

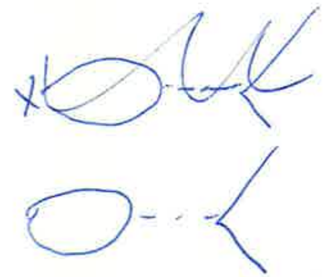
Note Boson fields are measurable → field theories as $\hbar \to 0$
Fermion " not " → particle theories as $\hbar \to 0$

Heitler states fermion field has no physical significance
Heisenberg says it cannot be measured since interaction involving electrons always involves quadratic terms (charges or currents)

* This is because N_k does not commute with H eg.
thus $a_k^\dagger a_k \cdot a_k \neq a_k a_k^\dagger a_k$.

† Particle density fluctuations actually arise in 2 ways: —

① vacuum polarization



This shows how

Knowing $N_{v_1} = 1$, we have to allow N_{v_2} N_{v_3} etc to be possible / also when $v_2 v_3$ .. are within $\hbar/mc$ of v_1 — this involves possibility of creating electrons — a truly relativistic effect.
(this due to Pauli principle) [— vac. polarization is an extra effect of Uehling

② Lamb-Darwin effect

Single electron couples to vacuum fluctuations of the photon field — produces wobble. This wobble leads to radiation then — recoiling electron extends over a distance $\hbar/mc$ but now we are only considering 1 electron which is possibly in different states

([Cf Weisskopf, R.M.P 21, 1949, p.305) & Welton (1948)]

Relativistic Quantum Field Theory

Due to the possibility of creating particles by supplying rest energy the concept of a localized particle becomes essentially blurred in relativistic theories of elementary particles †

In fact particle densities become spread over the order of the range ħ/mc.

In addition for the fields interacting with the particles we have the fact of vacuum fluctuations in field amplitudes even when no particles are present — this arises naturally in a field context from the q. theory of the harmonic oscillator. *

These 2 effects, fluctuations of field strength and of particle density, are what produce the so-called radiative corrections in the Q.E.D. The 2 effects are closely linked since the density fluctuation → [~~vacuum polarization~~] (or Tamm-Dancoff coupling) → coupling with vacuum field oscillators (½)

In relativistic field theory we recognize at the outset that the description of 1 real particle involves the possibility of a many-particle problem. Hence the 2nd quantized formulation is essential in relativistic theories.

We do not assume field amplitudes are observable (they may not even be hermitian) but we introduce the concept of a complete set of localized observables "tied" to a space-like surface, every number commuting.

We are interested in energy density, momentum density, charge density, (integrated over small volume elements) We assume classical formulae for these quantities, e.g. as generators of infinitesimal canonical transformations of the classical system under symmetry operations - space

are then interpreted as infinitesimal generators of the associated unitary transformations in the Hilbert space of Q. Mechanics. (cf Schwinger 1951 e.g.) the field variables are interpreted as operators subject to quantization rules, commutators or anticommutators, but there are no extra assumptions and really have nothing to do with 1st quantization, except in a heuristic sense for fields in which the amplitudes are measurable (ex electromagnetic field)

The two features peculiar to rel. Q.F. theory which arise from the union of relativity which allow the possibility of particles to be materialized out of Q. mechanics which provides the energy necessary for this materialization by fluctuations having their source in the uncertainty principle.

* reduction to log. self-energy is due essentially to the smearing of the charge density $\sim \hbar/mc$ in hole theory (effect of Pauli Principle — not vacuum polarization)

The fluctuation phenomena described led to the notion of the vacuum being revived — Indeed relativity had started off by abolishing the vacuum. (ether)
But now the combination of relativity with Q. mechanics led to ~~the~~ "renovation" of the vacuum.
This led immediately to the problem of electron — self-energy due to coupling with virtual transverse photons (Heisenberg, Pauli (1929, 1930), Oppenheimer (1930), Rosenfeld (1931) showed a quadratic divergence) even in hole — theory Weisskopf (1934) found a logarithmic divergence) *
which for vacuum polarization Uehling (1935) and Dirac (1934), Heisenberg (1934), Weisskopf (1936) found a quadratic divergence
Uehling's calculation showed how Coulomb law of force between 2 charges is modified

P.T.O

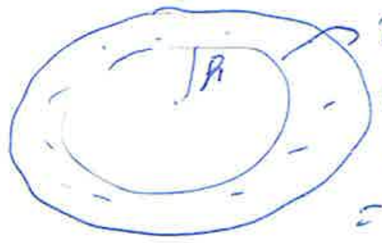
by polarization of the vacuum in the
neighbourhood ($r\ \hbar/mc$) of the charge
(e.g. due to local creation of e^+e^- pairs)

These divergences arise essentially from summation over intermediate states (energies) of the virtual photon

as in $$\Delta E = \sum_n \frac{\langle 0|H'|n\rangle\langle n|H'|0\rangle}{E_0 - E_n}$$

the $\sum_n \to \int_k k^2 dk$ since no. of translation states between k, $k+dk$ is $\sim 4\pi k^2 dk$

also $E_n \sim k$



vol. of spherical shell $= 4\pi k^2 dk$

Hence → quadratic divergence.

Lamb Shift 5a Lecture IV 1

Self-energy:

classical electron : ∞ coulomb self-energy bringing 2 charges into coincidence
similarly for a sphere.

The Lorentz self-force

wrong transformation properties of self-energy

— non-electromagnetic forces — the self-stress problem

— also what holds charge distribution from flying apart.

Attempts at removing infinities in classical electrodynamics summarized in Schweber.

In Q. theory divergences are of a different kind & are more serious

① Transverse self-energy of a stationary electron $\Delta E = \sum_n \frac{\langle 0|H'|n\rangle\langle n|H'|0\rangle}{E_0 - E_n}$

Since $\langle\ \rangle \propto \frac{1}{\sqrt{k}}$ and E_n an [illegible] state. $\rightarrow \int \frac{k^2 dk}{k} \rightarrow$ quadratic divergence

(Heisenberg, Pauli (1929, 30), Oppenheimer (1930)
Waller (1930) Rosenfeld (1931))

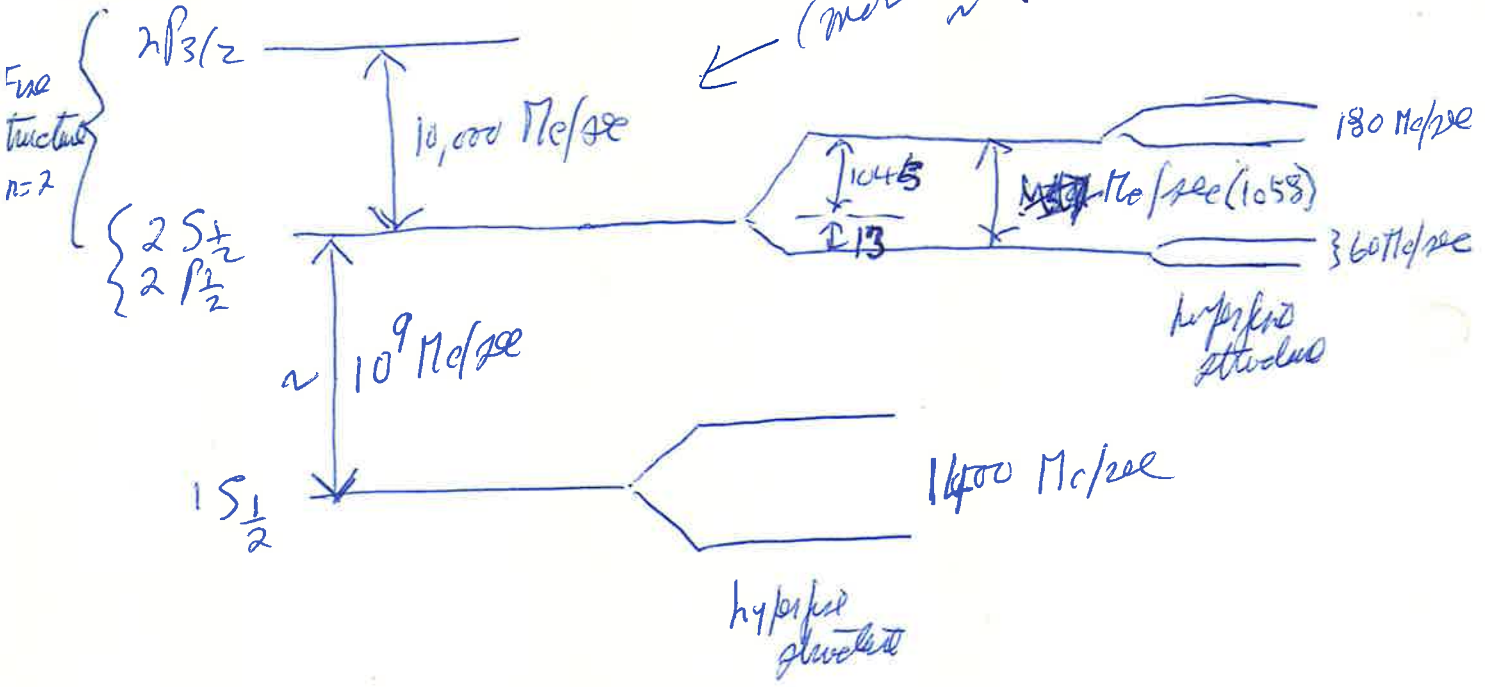
(2) In hole theory divergence is reduced to logarithmic by the spreading of the charge due to the Pauli Principle (Weisskopf (1934)) but it is not eliminated.

(3) The phenomenon of vacuum polarization which makes coulomb law between charges after leads to infinities (Uehling (1935))

Source of difficulties traced by Heisenberg to sharp separation of relativistic light cone. leads to singular commutation rules.

Note in Q. theory higher order process all divergent — contrast with classical case.

Hydrogen spectrum (after Dirac)



(more accurately ~ 11,000 Mc/sec)
Fine structure n=2
$2P_{3/2}$
10,000 Mc/sec
$2S_{\frac{1}{2}}$
$2P_{\frac{1}{2}}$
1045
13
~~1040~~ Mc/sec (1058)
180 Mc/sec
} 60 Mc/sec
hyperfine structure
~ 10^9 Mc/sec
$1S_{\frac{1}{2}}$
1400 Mc/sec
hyperfine structure

N.B. Lamb shift of S-levels ~ $1/n^3$, so decreases rapidly as n increases.

hyperfine structure of levels is ~ 1420 Mc/sec. for ground state but only ~ 200 Mc/sec for $n = 2$ due to smallness of nuclear magnetic moment. — due to large mass of nucleon relative to electron.

h.f.s. is a known effect which is too small to account for the gross Lamb shift — but theory of Lamb shift has to be corrected for h.f.s.

Lamb shift Small separation $2S_{\frac{1}{2}}$, $2P_{\frac{1}{2}}$ levels in hydrogen which coincide in Dirac theory.

S-states are raised by about 1000 Mc/sec. This is equivalent to a change in wavelength in the hydrogen spectrum of about .0003 A.U.

$$\Big[\ \lambda\nu = c \quad \therefore \quad \lambda d\nu = \nu d\lambda = \cancel{\nu c/\Delta\lambda}$$

$$\therefore \frac{d\lambda}{\lambda} = \frac{d\nu}{\nu} = \frac{c}{\Delta\lambda\,\nu} = \frac{\lambda}{\Delta\lambda}$$

$$\Delta\lambda = \frac{3\times10^{10}}{10^{9}} = 30 \text{ cm.}$$

$$\lambda = 10^{-5}\text{ cm } (= 1000\text{ A.U.}) \quad \therefore d\lambda = \frac{\lambda^2}{\Delta\lambda} = \frac{10^{-10}}{30} = .3\times10^{-3}\text{ A.U} = 0.0003\text{ A.U}\ \Big]$$

So separation of lines is small compared with hyperfine structure (down to .1 A.U)

Anomaly in hydrogen spectrum first analysed by Pasternack (1938) from spectroscopic evidence

Why it is not due to the nucleus

1.) It can be accounted for by an extension of charge of radius of electron of order of $\hbar/mc$ — but nucleus is 1/100 smaller than this from scattering experiments

Anomalies would turn up at $\approx .4\, mc^2$
$= 200$ keV.

[when $\lambda = \hbar/p \approx \hbar/m_0 c$, $p = mv$

So $\underbrace{mv = m_0 c} = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$

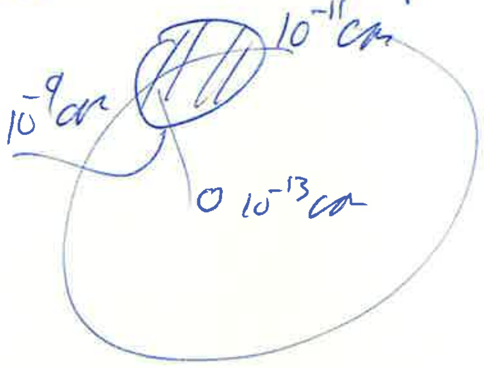
$\therefore v^2/c^2 = 1 - v^2/c^2$ or $v^2/c^2 = 1/2$

$\sqrt{1-v^2/c^2} = 1/\sqrt{2}$.

K.E. electron $= \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) m_0 c^2$]

Note, however, if electron has speed $\sim \hbar/m_0 c$ this will suffer a Lorentz contraction so spread remains of $\sim$ of de Broglie wavelength. 10^{-13} cm will not be reached for λ until K.E. is around 70 MeV — then form factors of nucleon will begin to show up — this is what is actually observed

Picture is thus



2. ~~2~~2) magnetic effect of nucleus is ruled out as ~~it affects the~~ [the effect does not appear to depend on J appreciably, i.e. the spin is not affected, and shift is greatest for S-states ~~which~~ have no orbital magnetic moment]
it gives too small an effect for $n=2$ levels.

It was suggested by Kramers that effect might be due to electron wobble, just prior to Lamb-Retherford experiment in 1947

Kramers has suggested dealing with ∞'s by renormalization.

Bethe took up Kramer's ideas & whole problem was discussed at Shelter Island Conference in 1947 — Oppenheimer, Schwinger & Weisskopf etc contributed greatly.

Bethe did a simple calculation of difference in self-energy of an electron in a bound state & a free state and obtained figure of 1040 for Lamb shift in reasonable agreement with Lamb's results.

Problem was to justify renormalization to all orders of perturbation

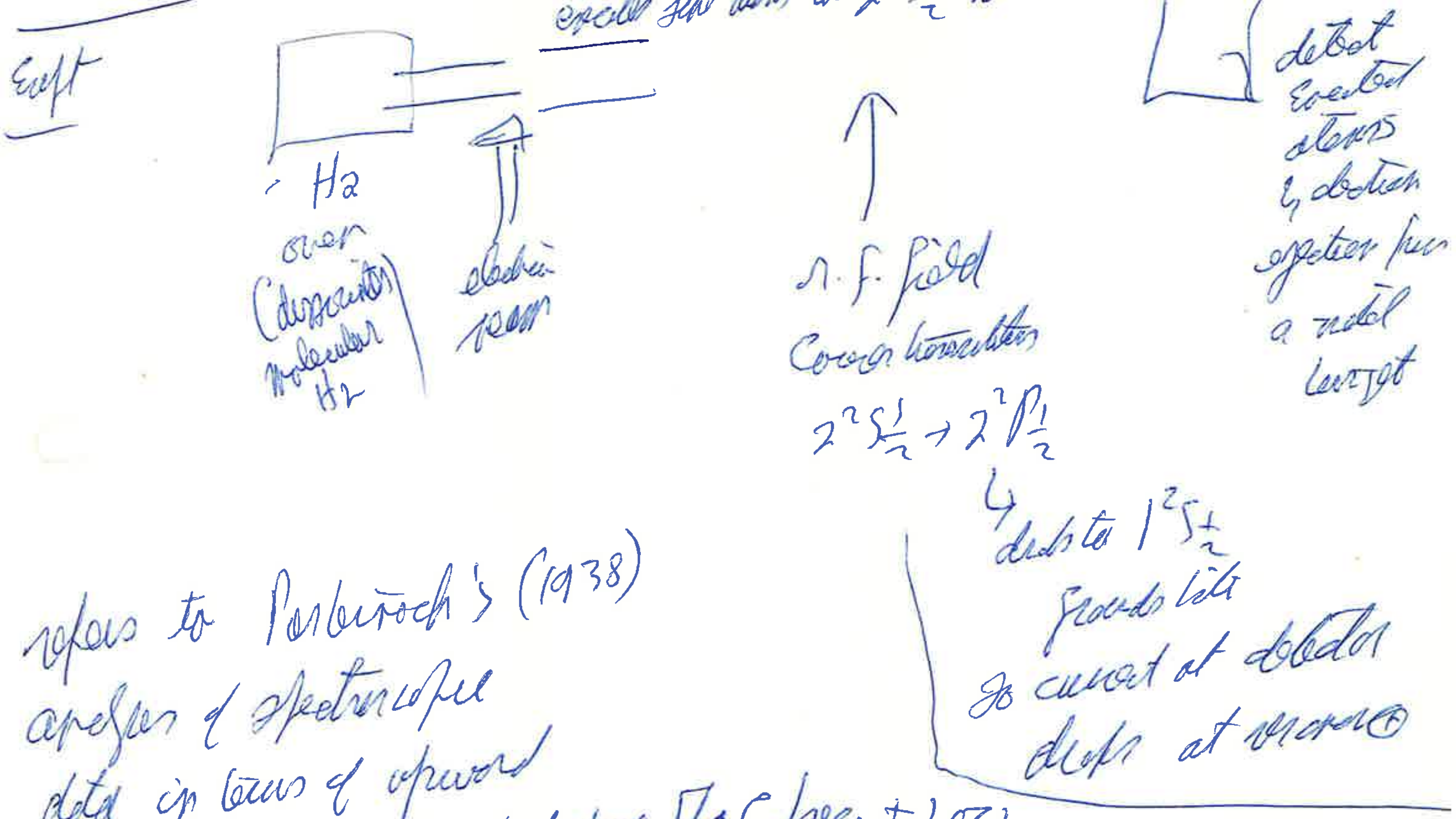
---

This led to covariant subtraction procedures being designed to avoid ambiguities in subtracting ∞ quantities.
This in turn depended on formulating field theory in an covariant way
— Tomonaga (1946) & Schwinger (1948-49) independently introduced the idea of replacing t by a space-like surface and the state-vector a functional instead of just a function of t.
Very involved calculations by Schwinger justified renormalization in 2nd order.

Then came Feynman's space-time approach, (1949) to be followed by Dyson derivation of the diagram techniques from the Schwinger-Tomonaga formalism. This was now forsaken & the Dyson-Wick (1950) analysis of the Feynman rules became the standard method approach — Renormalization to all orders of perturbation demonstrated by Dyson in ~~1950~~ 1949

Lamb 1947. P.R. 72 (241)
Retherford

Expt


H2 oven (dissociates molecular H2)
electron beam

excites some atoms to $2^2S_{\frac12}$ metastable state.

slow transition to $2^2P_{\frac12}$ due to low ΔE (?)

detect excited atoms by ejection from a metal target

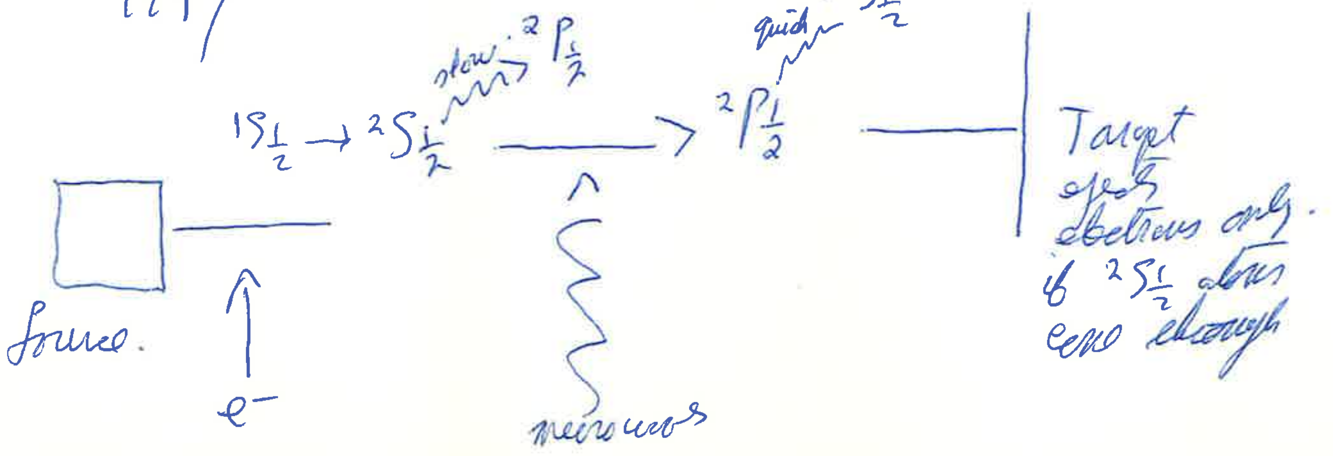
R.F. field causes transition $2^2S_{\frac12} \to 2^2P_{\frac12}$

decays to $1^2S_{\frac12}$ ground state

so current at detector drops at resonance

refers to Pasternack's (1938) analysis of spectroscopic data in terms of upward shift of S level by about 1000 Mg C./sec. ± 100

Experiments discussed at Conference on Foundations of Q. Mechanics at Shelter Island on Jun 1-3 1947


$^1S_{\frac12} \to {}^2S_{\frac12}$
slow
$^2P_{\frac12}$
$^2P_{\frac12}$
quick
$^1S_{\frac12}$
Source.
e^-
microwaves

Target ejects electrons only if $^2S_{\frac12}$ atoms come through

B

Lamb's experiments were continued in period 1947-1953 & v. accurate result achieved was.

$$\Delta E = 1057.77 \pm 0.10 \text{ Mc/sec.}$$

Experiment repeated by Robiscoe & Shyn (1968) and Lamb's analysis of his experiment was slightly revised. (Robiscoe & Shyn (1970))

Best final result is

$$1057.88 \pm 0.06 \text{ Mc/sec.}$$

Lamb's experimental value has survived v. well.

Theory developed:	Bethe (1947)	1040 Mc/sec
	Kroll & Lamb (1949) French & Weisskopf "	1052.
	Salpeter (1953). (summary of work by Karplus, Klein, Schwinger, Bethe, Feynman, Baranger, Dyson, Brenner, Bersohn, Kroll)	1057.2
Then	Fried & Yennie (1960) Layzer "	1057.70 ± 0.15

Latest value quoted in Lautrup, Peterman & Rafael 9 (1972)

is 1057.911 ± 0.012

Magnetic moment of electron

Theory 1st calculated by Schwinger (1948) $\Delta\mu = (0.001161)\ \underset{\text{Bohr magneton}}{\overset{\downarrow}{e\hbar/2mc}}$

to 2nd order

Karplus, Kroll (1950) $\Delta\mu = 0.001145$

4th order

error corrected by Sommerfield, Peterman, Kroll } (1957) $\Delta\mu = 0.0011596$

(closer to Schwinger's 2nd order)

Latest result to 6th order (α^3) is

$\Delta\mu =$ ~~0.00115964l~~

~~± 0.0000,0000~~3

0.00 11596554 Bohr magnetons

± 33

Experiment anomaly discovered by

Nafe, Nelson & Rabi (1947) using microwave analysis of hyperfine structure

Breit (1947) suggested explanation in terms of anomalous moment (intrinsic Pauli-type)

Kusch & Foley (1947) used resonance techniques on Zeeman levels

By 195 2 value of $\Delta\mu = 0.001146 \pm 0.000012$ had been achieved, v. close to ~~Kroll~~. Karplus & Kroll's answers and

but later experiments used not bound electrons, but free electrons

using rotation of polarization of beam of electrons by Larmor precession in a magnetic field initiated by Louisell, Pidd & Crane in 1954

This approach reached perfection in Wesley & Rich (1970) & unpublished

who found $\Delta\mu = 0.0011596577 \pm 35$ Bohr magnetons

(latest value quoted in Lautrup et al 1972)

Anomalous moment of the muon

theory similar to electron but measured mass of muon. spoils some approximations slightly greater uncertainty in prediction (about 15 ppm)

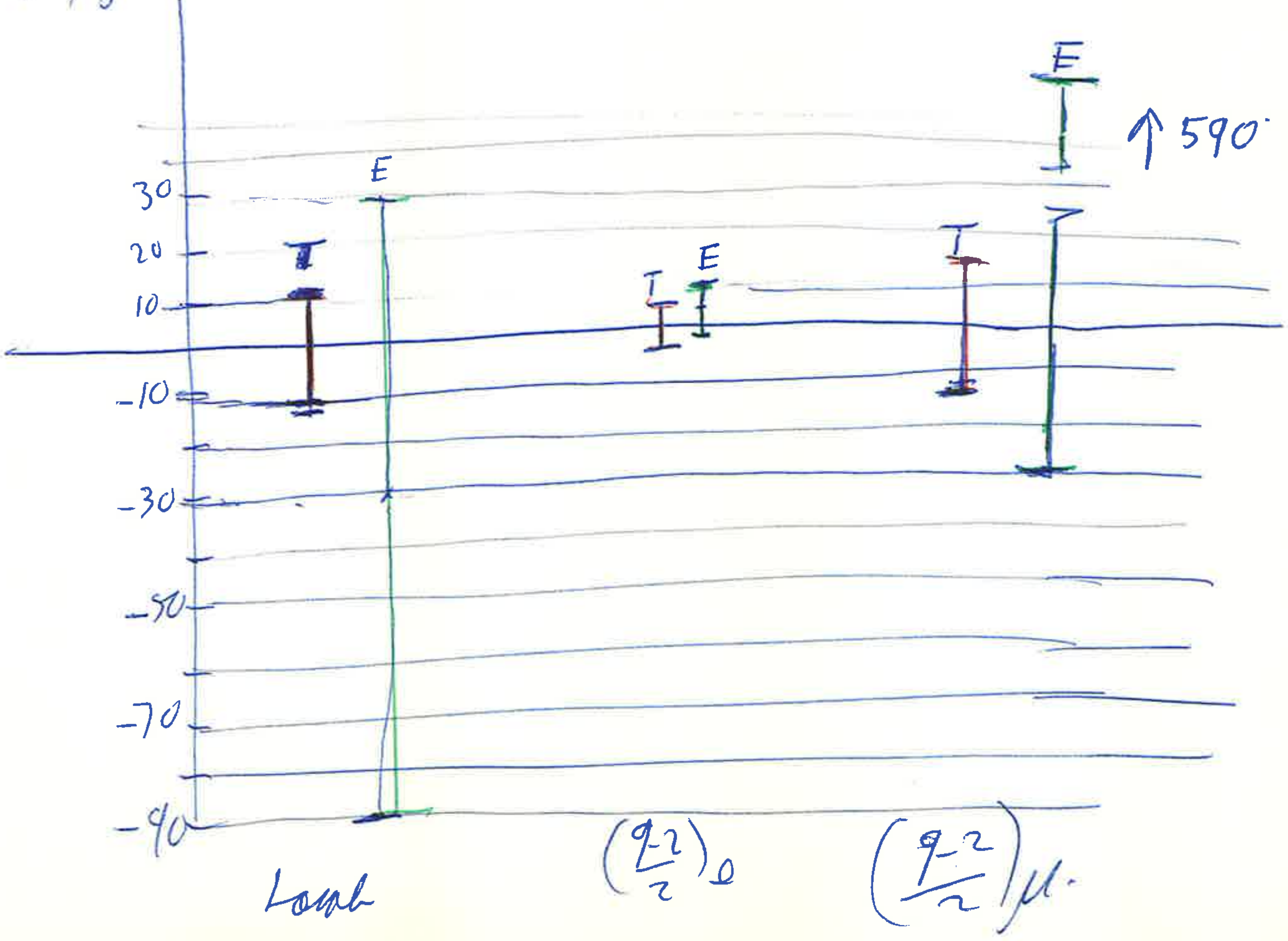
Experiments using the muon storage ring carried out at CERN by Bailey (1968) Expt. error 300 ppm but appears too high, just outside limit of error experiment now being repeated.

- essentially measures Larmor precession of muon spin relative to orbital cyclotron motion of the muon in a magnetic field (storage ring).

We can summarize comparison theory & Expt. as follows:

accuracy ppm	Theory	Expt	Δ(Exp−Theory)
Lamb	± 12	± 60	− 30
$\left(\frac{g-2}{2}\right)_e$	± 3	± 3	+ 2
$\left(\frac{g-2}{2}\right)_\mu$	± 15	± 310	+ 280

7th figure



Lecture V (Mention Lerner, Wright latest calc. on $(g-2)/2$ discrep. down 3 p.p.m., $n \to 5$ ppm. 1

1) Dyson's 1952 paper on Asymptotic character of perturbation expansion. cf also soluble models

2.) ~~Mention~~ infra-red divergencies

$$\sigma_n = I_0\, W(k_{max}, k_{min})^n\, e^{-W(K, k_{min})}/n!$$

perturbation theory gives

$W(k_1, k_2) = \text{const.} \ln(k_1/k_2)$

$$\sigma = \sigma_0 + \sigma_1 \simeq I_0\left(1 - W(K, k_{max})\right)$$
$$= I_0(1-\delta)$$

independent of k_{min}.

$$\sigma = \sum_0^\infty \sigma_n = I_0\, e^{-W(K, k_{max})}$$
$$\equiv I_0\, e^{-\delta}$$

If $k_{max} \simeq K$, then $\sigma_n = I_0\, W^n e^{-W}/n!$

W diverges like $\ln(1/k_{min})$.

This is a Poisson distribution and $\bar{n} = W \to \infty$

But $\sigma = \Sigma \sigma_n \to I_0$.

although $\sigma_n \to 0$ for any finite n.

Essentially e^{-W} is effect of infra-red divergence in virtual processes, $W^n/n!$ is effect of real radiation

cf. Bloch & Nordsieck (1937), Glauber (1951), Jauch (1954), Yennie Frautschi & Suura (1961)

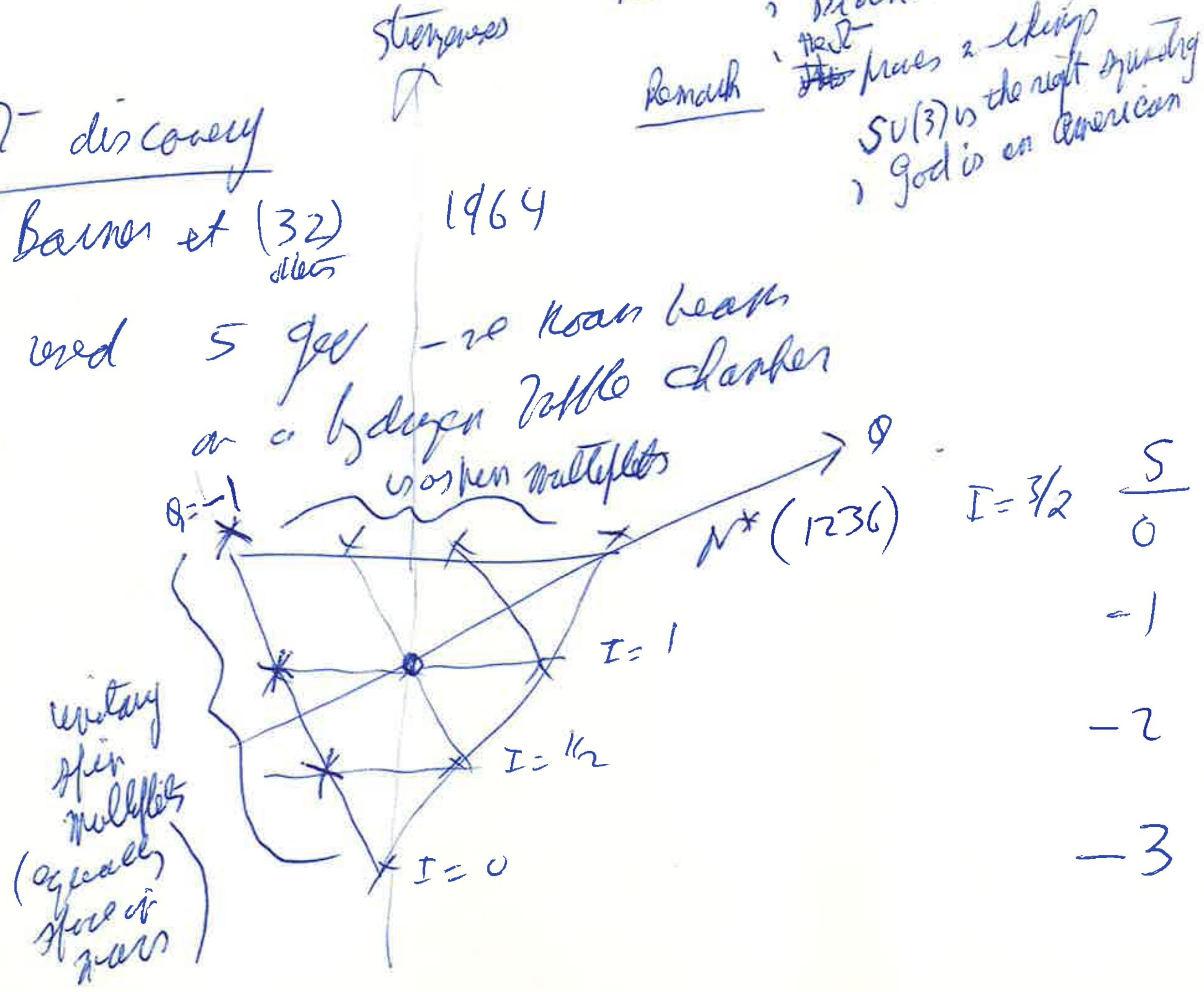
Rival teams at CERN
, Brookhaven

Remark 'the Ω ~~the~~ has a charge
SU(3) is the right symmetry
, God is an American'

Ω^- discovery

Barnes et (32) 1964
authors

used 5 GeV -ve kaon beam
on a hydrogen bubble chamber



strangeness
isospin multiplets
Q = -1
Q
$N^*(1236)$
I = 3/2
I = 1
I = 1/2
I = 0
unitary spin multiplets (equally spaced in mass)
S
0
-1
-2
-3

3) Other methods of theory development:

a.) modification, eg Klein-Gordon from Maxwell

$\propto e^{-\mu r}/r$ from $\propto 1/r$

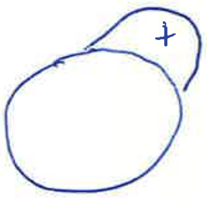
$$(\nabla^2-\mu^2)\phi = -4\pi\,\delta(\underset{\sim}{r}) \qquad \nabla^2\phi = -4\pi\,\delta(\underset{\sim}{r})$$

or V-A interaction in β-decay from parity conserving interaction

b.) analogical extension

eg strangeness Q.No. as a new sort of charge (hypercharge)

or SU2 → SU3



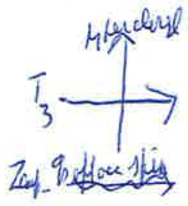
unitary unimodular R(3) is homomorphic ~~with~~ to SU(2) ~~the rotation group R(3)~~

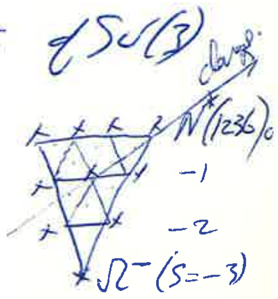
(N.B. ~~G~~ SL2,C Lorentz group L(4) ~SL(2))

Leads to multiplet structure for mesons, ~~approx~~ baryons from the basic 3 dimensional "quark" representations of SU(3)

thus $3\otimes\bar{3} = 8\oplus 1$

$3\otimes 3\otimes 3 = 8 \oplus 8 \oplus 10 \oplus 1$





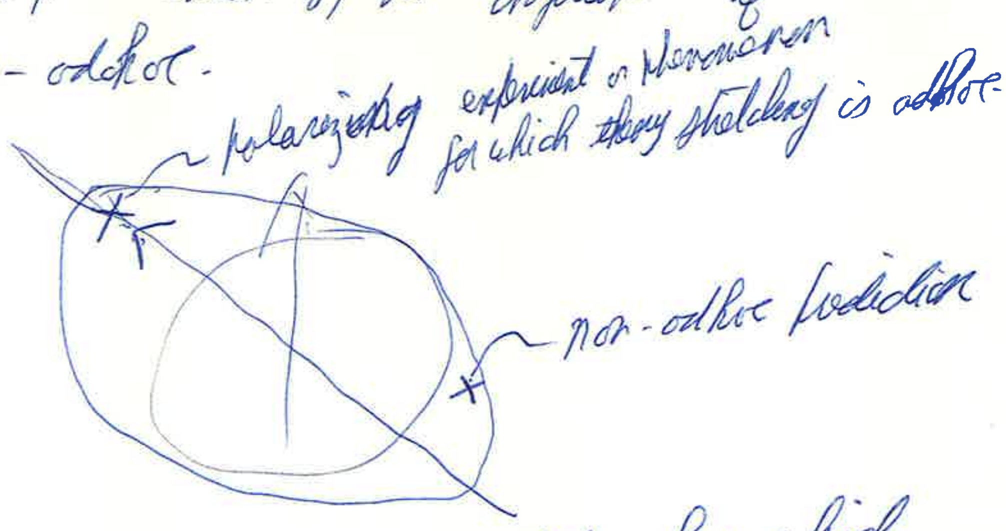
④ Chronological element is very important, cf. Kaon regeneration (1955)

Ω^- discovery in 1964

Barnes et al (32 others) 1964

$$\underset{-1}{K^-} + \underset{0}{p} \to \underset{-3}{\Omega^-} + \underset{+1}{K^+} + \underset{+1}{K^0} \quad S \to$$

For directional shift chronology not important if theory is non-adhoc.


polarizing experiment or phenomenon for which theory stretching is adhoc.
non-adhoc prediction

This is exemplified by Lamb shift for which polarizing phenomenon is production of particles (theory of radiation) — Lamb-shift is a non-adhoc explanation of an already known effect — cf. general relativity → explanation of motion of perihelion of Mercury.

Counterexample to Lakatos' original definition of a progressive research programme in terms of chronological novelty.

(5) Experiments can be dictated by theory

Ex's (1) Kaon regeneration (1955)
(2) Ω^- (1964)
(3) $\bar{p}$ (1955)
(4) ν (1956)

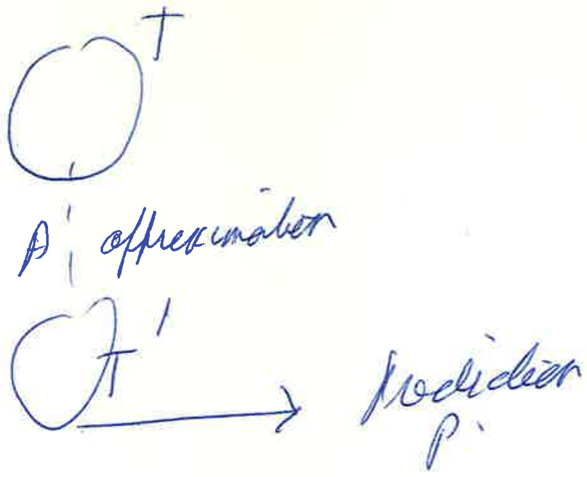
or theory can precede experiment but not influence the experimental discovery

Ex (1) e^+ (1932)
(2) muon (1936)

(6) Importance of quantitative prediction

cf. Hylleraas (1930) ground state of He (^1S)
) Lamb-shift
) astronomical predictions.

the Confutation gap



Modus Tollens directed
at A not T

then $\sim P \rightarrow \sim T' \rightarrow \sim(T \wedge A)$

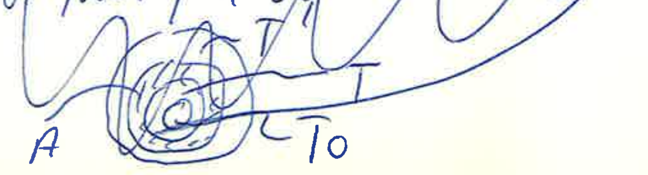
$\leftrightarrow (\sim T) \vee (\sim A)$

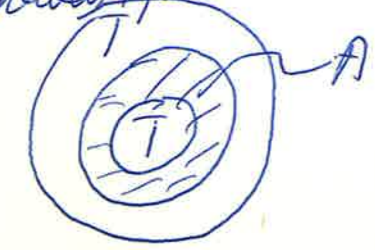
choose $\sim A$.

then A is part of the protective belt & positive heuristic of a research programme, proper often consists in making different mathematical approximations — often referred to in physics as models. (new sense of the word model)

T includes the hard core & negative heuristic

or possibly a double belt model.





Theory development

Lakatos is right to stress two aspects:—

Tenacity of theories → conventionalism
mutability ~~Transience~~ Flexibility of theories → falsificationism

These two aspects must be combined in any adequate rational reconstruction of the history of Science.

This has lead to Lakatos Methodology of Research Programmes, thus

neo justificationism = probabilism

most Physicists Bondi Redhead

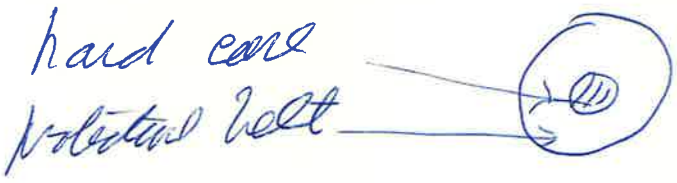
Verificationism (Carnap)
Conventionalism (Duhem-Quine)
Falsificationism
- ~~naive~~ Dogmatic (Weyl, Braithwaite)
- naive methodological (Popper$_1$)
- Sophisticated methodological (Popper$_2$)

Research Programmism Lakatos.

We attempt to analyse some case-studies in the Lakatos methodology, & also stress non-adhocness as a criterion of progressive problem shift rather than temporal novelty.

→ the unit of variation in our own view
is a micro research programme with
hard-core & protective belt rather
than a single theory

Lakatos analyses these



The negative heuristic of the ~~[illegible]~~ Programme (series of theories) are methodological rules saying what paths of research to avoid, i.e. not to alter the hard core.

The positive heuristic are rules telling ~~to~~ us that the protective belt can be altered leading to the progressive or degenerating problem shift

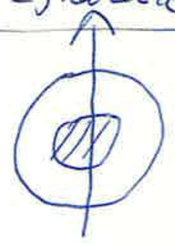
We can extend the analyses in more detail ~~as follows~~ as shown diagrammatically below.

Notice how distinction between revolutions & normal science becomes blurred on analyses — normal science really for a few structures of micro revolutions each with its own hard core & protective belt - this leads to Toulmen's concept of an evolutionary theory of scientific change - variation & selection which is really ~~quite close~~ identical with to Popper's account in 'objective knowledge' — an evolutionary approach. *

Development of a ~~Theory~~ Field-Theoretic Programme

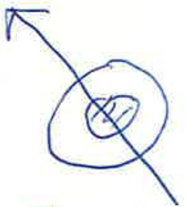
classical
N-particle theory

paradigm shift →



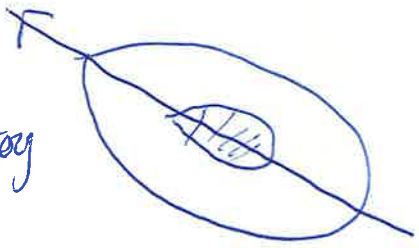
N-particle Schrödinger Eqn

directional shift →



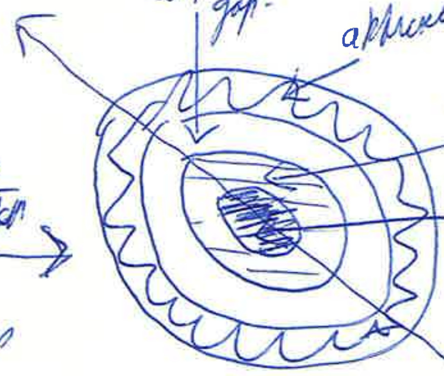
2nd quantization
formulation

stretching theory →



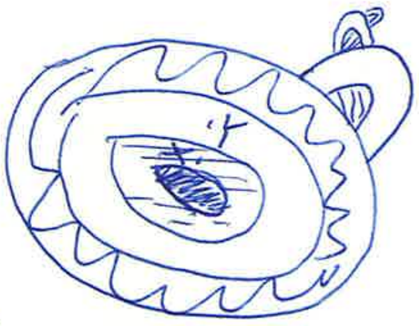
variable No of particles

Testable
consequences
Calculation →
gap
~~model~~
predictions


computation gap.
approximate theory
protective belt
hard core

approximation schemes
for field theories

problem shift according →
to the positive heuristic
of the Programme



either by { alteration at X → new theory,
alteration at Y → new model of old theory

Po

Complete with discussion of correspondence
Principle in theories and read
quotations of Bernstein & [illegible]

Rational reconstructions are
judged against accepted 'basic value
judgments' of the Scientific Elite. (Lakatos 1971)

Correspondence works in general thus!
L & S in corresponds if L can be
rationally reconstructed as a modification
or extension of some formulation of S.
On this view paradigm shifts in
our terminology lead to incommensurable theories
no correspondence in the sense of a rational
reconstruction.

Note: the fact that L explains ~~all~~ (potentially) all that S explains means that S must be capable of representation within L, with suitable rules of correspondence establishing the appropriate translation between L and S but this does not mean that the rational historical reconstruction ~~in the~~ (rational in the sense of Lakatos' meta criterion) of the development of L can be expressed in terms of correspondence. * In particular the historical sequence must be right as in capture + scattering of light both theories were advanced before correspondence was proved — same is true of Quantum Mechanics, although correspondence arguments were important in 'guiding' the new theory

EKS.

Aristotle → Galileo, Newton (what is natural? new metaphysics)

Ptolemy → Copernicus (External factor vital)

Corpuscular → wave theory of light (correspondence discovered after both theories advanced)

Galilean relativity → Einsteinian relativity in Mechanics
(based on a new invariant constant)

classical mechanics → Q. Mechanics.
(wave mechanics à la de Broglie is an example however)

These can be represented as examples of correspondence only by a distortion of the rational reconstruction of the history of Science *

Eg (1) Hamilton-Jacobi Eq.
→ wave Eq. as
correspondence of corpuscular → wave theory of light

(2) Boolean Lattice of Propositions
→ Non-Boolean lattice (i.e. non-distributive) of propositions
as correspondence of classical, Q. mechanics
(Birkhoff, Von-Neumann)

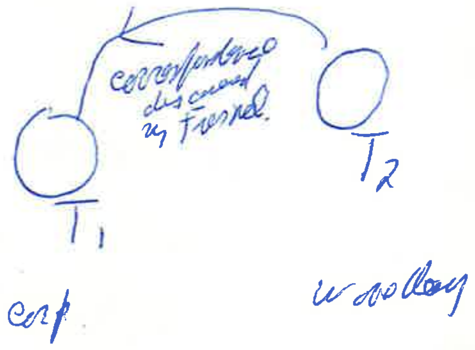
N.B. Thermodynamics was first theory of interaction of heat, work as a const.
p.t.o

cannot be represented as a reformulation of an existing theories.

But other advances in classical physics appear to be examples of modification or analogical extension.

Ex of corpusc. + wave theory of light

Initially 2 groups of phenomena seemed to require both theories :- rectilinear prop.) diffraction phenomena.


correspondence discovered by Fresnel.
T_1
T_2
corp.
w. theory

Lecture VI Schwinger, Tomonaga, Feynman, Dyson

The problem of the unambiguous subtraction of infinities in Q. field theory depended on 2 advances:—

① a manifestly covariant formulation of relativistic Q. field theory

as the usual S. Eqn $i\hbar \frac{\partial \underbrace{\Psi}_{\text{state vector}}}{\partial t} = \int d^3x\, \underbrace{H(x)}_{\text{Hamiltonian density}} \cdot \Psi$

must be given a new invariant formulation

this was achieved in 2-ways

1) (1948) Schwinger } independently (developed invariant subtraction methods by) generalizing
Tomonaga } the multiple time formalism of Dirac Fock, Podolsky (1932)

so that Ψ is defined as a functional of a space-like surface Ψ[σ]

the S. Eq. is re-written as the so-called Schwinger Tomonaga Eqn

$$i\hbar \frac{\delta \Psi[\sigma]}{\delta \sigma(x)} = H_I(x)\, \Psi[\sigma]$$ (interaction representation)

1943 etc in Japanese translated into English in 1946 in Prog. Theor. Phys. — drew attention to this work in Phys. Rev. in 1948

[Sch. acknowledges Tomonaga's (1946) — invariant subtraction methods developed independently by Sch & Tom.]

1965 Nobel Prize

→ Tomonaga
Schwinger
Feynman }

1.) Feynman introduced his space-time approach to Q. theory as replacement for the familiar Hamiltonian approach. (1948)

Dyson showed that Feynman's methods could be derived by ~~standard~~ suitably manipulating the Schwinger- formalism (1949)

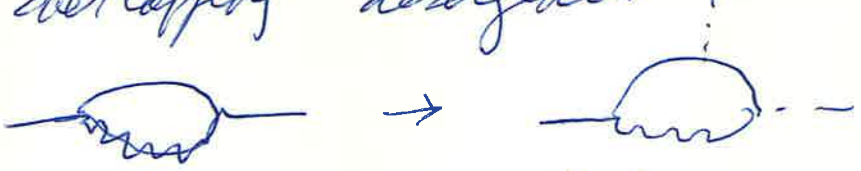
and then proceeded to implement the covariant subtraction procedure using the

(2.) The great computational advances made by Feynman using his diagrams (1949) to classify all the high order virtual transitions & to write down the associated Matrix element by inspection.

This was exploited by Dyson in his proof of the renormalizability of spinor electrodynamics (1949)

Dyson's proof was completed in 1951 by

a.) Ward using device of dummy lines to avoid overlapping divergencies.

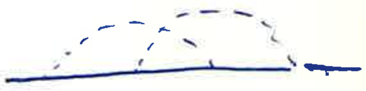


b.) Direct approach to overlapping divergences by Salam.

Difficulty mainly due to lack of notation for handling structure of sub-integrations in complicated multiple integrals

* "An adequate notation is one which is "both concise & intelligible" ~~comprehensible~~ to at ~~at~~ least two ~~people~~ persons ~~one~~ of whom one ~~whom~~ may be its author".

overlapping divergence



Dyson's method was to define primitive divergent
→ finite sol if any line is opened (not integrated over)

then classify possible primitive divergent in terms of the ~~no of~~ structure of external lines > no of vertices

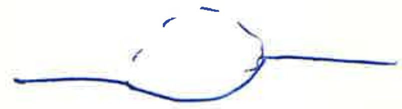
$$D = \text{const} - N(\text{external lines}) + N(\text{vertices}) \geq 0 .$$

(necessary condition but not sufficient) n D → convergence

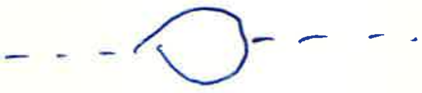
for renormalizable theories no N(vertices) term present. ⇒ only low no of external lines gives divergence.

3 primitive divergences

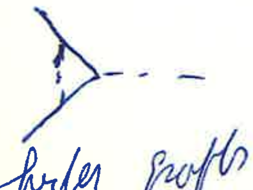
(a) electron self-energy



(b) photon self-energy or vacuum polarization



(c) vertex part

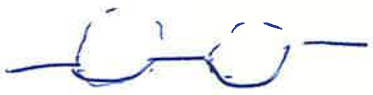


> higher graphs

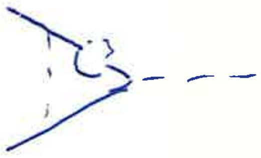
Then define irreducible divergent graphs by insertions of primitive divergents in the lines or at the vertices of primitive divergents.

3 possibilities

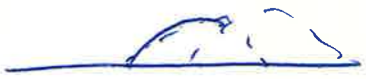
non-overlapping ~~separation~~



inclusion or containment



overlap



Then define subtraction procedure by subtracting off divergences in the primitives in an invariant manner leaving convergent results essentially by cancellation between 2 ∞ quantities — cf conditionally convergent series $1 - 1/2 + 1/3 - 1/4 - - \to \ln 2$. or any other value

(Dirichlet's theorem does not hold in such a case)

Deal with containment-type irreducibles by removing divergences starting from the innermost.

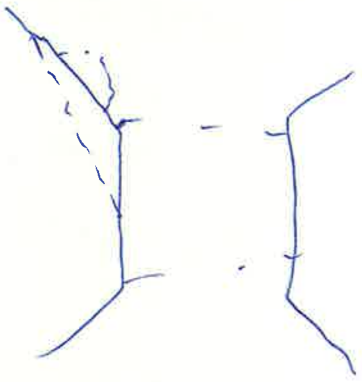
cf. effect of all vertex parts

is $e\Gamma_\mu = eZ_1^{-1}\,\Gamma_{\mu 1}(e_1) = e_1\Gamma_{\mu 1}(e_1)$

↑ finite part $\quad e_1 = Z_1^{-1}e$

— so far as vertex divergences are concerned.

as in



Subtraction procedure is then shown to be equivalent to renormalization essentially by showing that finite parts thus defined are really just ∞ multiples of the the unsubtracted ~~the~~ integrals and these multiples can be combined to renormalize the charge of the electron.

Mass renormalization is by Counter terms for the linearly divergent part of the electron self-energy. (which actually turns out to be logarithmically divergent)

How manifest covariance enables ambiguities in subtraction procedure to be eliminated 1

ex

Consider $I = \int_R f(x)\,dx$.

Then from a moving frame

$$I' = \int_{R'} f'(x)\,dx.$$

$$f'(x) = \Lambda f(L^{-1}x)$$

Suppose R is whole of space-time

Then $I' = \int_R f'(x)\,dx$.

Then it is easy to show that $I' = \Lambda \int_R f(x)\,dx = \Lambda I$.

Suppose we have a scalar integrand $\Lambda = 1$

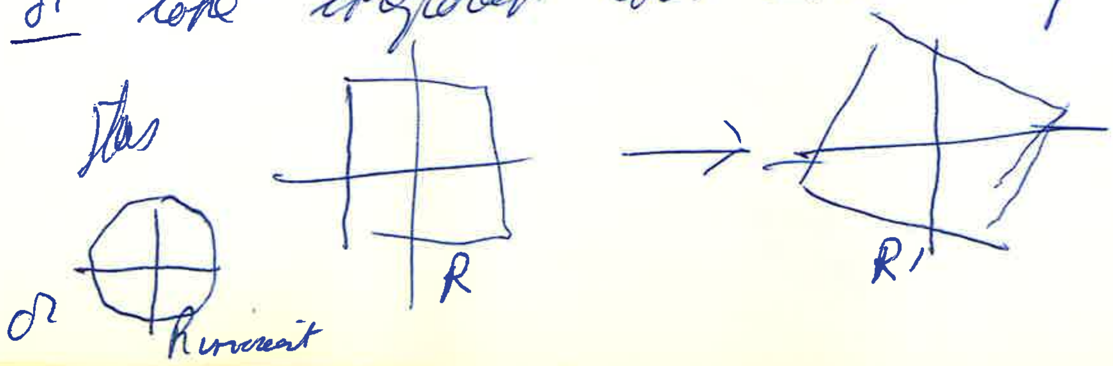
Then $I' = I$, a relativistic invariant.

Now suppose R is not whole of space-time.

Then $I' = I$ only if for I' we integrate over R'

or take integration over invariant region ([illegible])

Thus



We can write

$$I = I_1 + g(\hat{n})$$

↑ term depends on orientation of R

This term is absent for covariant theory, can be made zero for a covariant theory by choosing $R' = R$ = invariant fn

~~If I(n) is a scalar invariant~~

I = the choice of $\hat{n}$ would be an arbitrary matter which would spoil Lorentz invariance by introducing a preferred "direction" in spacetime

But suppose we evaluate $\int_{\text{all space}}$ for I' over R.

$$\text{then } I'' = \int_R f'(x)\,dx \neq I \qquad — ①$$

~~For a convergent theory the move from.~~

$$\text{But } I' = \int_{R_{\text{invariant}}} f'(x)\,dx = I \qquad — ②$$

For a convergent theory the move from R to $R_{\text{invariant}}$ makes no difference and $I'' = I$ just as $I' = I$, but for ~~divergent~~ conditionally convergent integrals the change can be quite vital — we must specify how the integration is to be carried out (over an invariant region) if we are to obtain the invariant result for the integral.

Consider reflection symmetry $x \to -x$ as a simple example.

Then $I = \int_R f(x)\,dx$

$$I' = \int_{R'} f'(x)\,dx \qquad \text{where } f'(x) = f(-x)$$

Since this is a scalar function

$$I = I'$$

Now suppose $f(x)$ is an odd function $f(-x) = -f(x)$

Then $I' = -\int_{R'} f(x)\,dx = \int_R f(x)\,dx$. — ①

Suppose ~~R as~~ we also consider

$$I'' = \int_R f'(x)\,dx = -\int_R f(x)\,dx.$$

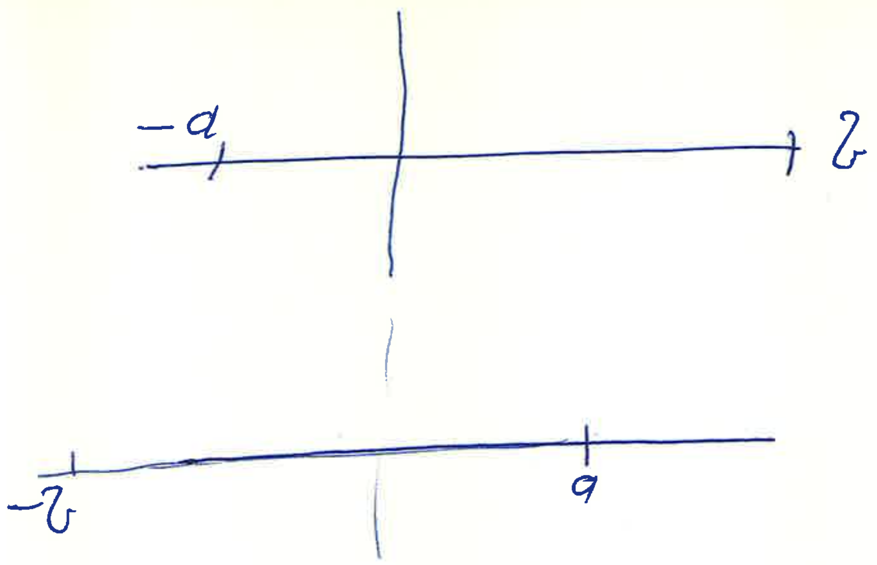
Suppose R, R' are the whole line, then we could naturally conclude from $I = I'$ that $\underline{I = 0}$

but if integral is not ~~divergent~~ convergent we must be careful how we proceed to the limits

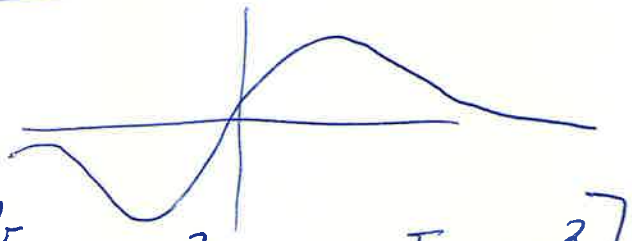
thus Take

$R = (-a, b)$

$R' = (-b, a)$



1st take $f(x) = xe^{-x^2}$



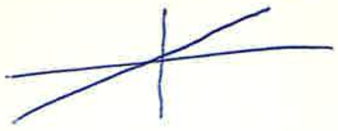
then $I = \int_{-a}^{b} xe^{-x^2} = \frac{1}{2}\left[-e^{-x^2}\right]_{-a}^{b}$

$= \frac{1}{2}\left[e^{-a^2} - e^{-b^2}\right] \to 0$ as $a \to \infty$, $b \to \infty$.

also $I'' = -\int_R f(x)dx = 0 = I$

So R can be used instead of R' and I'' and I' have same value $= I$.

But now take $f(x) = x$



$$I = \int_{-a}^{b} x\,dx = \tfrac{1}{2}(b^2 - a^2) = \tfrac{1}{2}(b-a)(b+a)$$

$$I' = -\int_{-b}^{a} x\,dx = \tfrac{1}{2}(b^2 - a^2) = I$$

But $I'' = -\int_{-a}^{b} x\,dx = -\tfrac{1}{2}(b^2 - a^2) = -I$.

So $I'' = -I$

Also for invariant interval (i.e. $a = b$)

$$I = 0$$

But for non-invariant interval take e.g.

$a = \overset{b - 1/b}{\cancel{(b + 1/b)}}$ and let $b \to \infty$

Then $I = \tfrac{1}{2}(b-a)(b+a) = \frac{1}{2b}(2b - 1/b)$

$\underline{\to 1 \text{ as } b \to \infty}$

So value of integral depends on how we approach the limits and it is only by ~~defining~~ specifying the use of an invariant interval in this problem that we secure the result $I = 0$ that would have to hold in a covariant theory.

our object in invariant subtraction is thus to define a procedure which agrees with what we would expect from a convergent theory

the factor e^{-x^2} in our example plays just the role of a convergence factor.

Quotes from Feynman

"It is as though a bombardier flying low over a road suddenly sees three roads and it only when two of them come together & disappear again that he realizes that he has simply passed over a long switchback in a single road"

I (1949)

" By forsaking the Hamiltonian method, the wedding of relativity & quantum mechanics can be accomplished most naturally"

II (1949)

$$K_0 \propto e^{\frac{(m)\left|\underset{\sim}{r}_2 - \underset{\sim}{r}_1\right|^2}{2\hbar(t_2-t_1)}} \times (t_2-t_1)^{-3/2}$$

The Feynman diagram

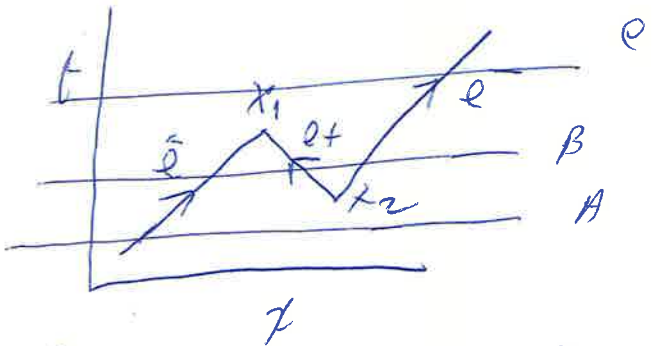
Feynman's idea was to unify the description of apparently unrelated processes:—

write
$$\psi(x_2, t_2) = \int d^3x_1 \, K(x_2 t_2 ; x_1 t_1) \, \psi(x_1, t_1)$$
$t_2 > t_1$

$$K(2,1) = K_0(2,1) - \frac{i}{\hbar} \int K_0(2,3) V(3) K(3,1) \, d^4x_3$$

exact
then all perturbation theory

Eg



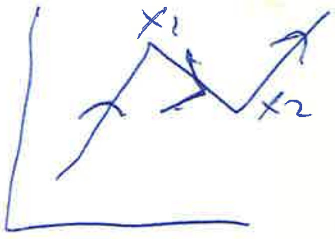
Hamiltonian view considers slices such as A B, C

At A there is one particle

B there are 3 particles

C is one again. we describe this by saying a pair of particles is created at x_2 and one of the pair thus created annihilates the incoming particle

Feynman draws the diagram thus:—



and says a single electron moves along a continuous trajectory in space-time — between x_1, x_2 it moves backwards in time with negative energy which is equivalent to moving forward in time with positive energy ($\psi(t) = e^{-i/\hbar \, Et} \, \psi(0)$)

To obtain the effect of scattering at any space-time point

Integral Eq. for Green's function

S. Eq. $\left(i\hbar\frac{\partial}{\partial t} - H_0\right)\psi = 0$

Solved by Green's function

$$G_0 = \left(i\hbar\frac{\partial}{\partial t} - H_0\right)^{-1}$$

(Inhomogeneous Eq. $G_0^{-1}\psi = F$ solved by $\psi = G_0 F$.) or homogeneous Eq. with inhomogeneous boundary conditions

Similarly for $\left(i\hbar\frac{\partial}{\partial t} - H\right)\psi$

Green's function is

$$G = \left(i\hbar\frac{\partial}{\partial t} - H\right)^{-1}$$ where $H = H_0 + V$

Then use ~~$\frac{1}{A-B} = \frac{1}{A} + \cdot 1$~~

$$\frac{1}{A} = \frac{1}{B} + \frac{1}{B}\cdot(B-A)\cdot\frac{1}{A}.$$

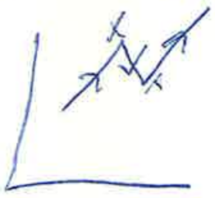
to give

$$\underline{G = G_0 + G_0 \cdot V \cdot G}$$ $\left(= \sum_{n=0}^{\infty}(G_0V)^n \cdot G_0\right.$

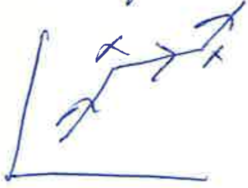
Soln : $(1 - G_0V)G = G_0$

or $G = (1 - G_0V)^{-1} \cdot G_0 = G_0 + G_0VG_0 + G_0VG_0VG_0 + G_0VG_0VG_0VG_0 + \cdots$

Feynman integrates over all x_1 x_2, thus placing processes like



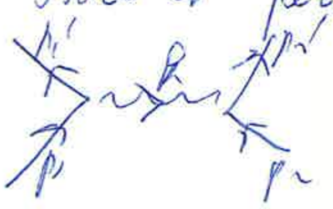
and

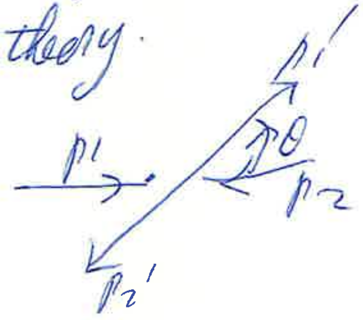


on an equal footing

In the usual form of the diagrams one works in momentum space and 4-momentum is then conserved at every vertex, but "internal" lines are not necessarily on the mass-shell, they correspond to virtual particle states in perturbation theory.

Ex





$$k = p_1 - p_1'$$

$$k^2 = (p_1 - p_1')^2 = p_1^2 + p_1'^2 - 2p_1 \cdot p_1' = 2M^2 - 2(E^2 - p^2\cos\theta)$$

$$= 2M^2 - 2(p^2 + M^2 - p^2\cos\theta)$$

$$= -2p^2(1 - \cos\theta) = -4p^2\sin^2\tfrac{1}{2}\theta$$

compare $k^2 = \mu^2$ for a real meson of mass μ.

Feynman diagrams enable self-energy effects to be readily evaluated in terms of Lorentz-covariant integrals

But the integrals diverge — divergences can be removed by renormalization $\left(\int_{0'} A'(x')\,dx' = \int_0 A(x)\,dx \right)$

$M = M'(m_0, \lambda) + M_0(m_0)$

Write $M = M'(m - \delta m, \lambda) + M_0(m - \delta m)$
now choose δm to cancel M'

thus we require $M = M'(m - \delta m, \lambda) + M_0(m - \delta m)$
$= M_0(m)$

Expand for small δm
gives $\delta m = \dfrac{M'(m, \lambda)}{\dfrac{\partial M'}{\partial m} + \dfrac{\partial M_0}{\partial m}}$

$\delta m \equiv \delta m(m, \lambda)$

eg electron 2nd order
$\delta m = \cancel{\frac{m e^2}{2\pi^2}(3\ell} \quad \frac{m\alpha}{2\pi}\left(3 \ln\left(\frac{1}{\lambda m}\right) + 3/4\right)$

This works for one diagram
we have to show cancellation applies
for all diagrams

Lorentz self force on classical electron $\quad \Gamma^s = -2/3\, \dot{v}/c^2 \cdot e^2/r_0$
$+ 2/3\, e^2/c^3\, \ddot{v}$

$\int [illegible]\, d\Gamma \approx a/r$
$\int [illegible]\, d\Gamma \approx H$

$\left(= \frac{d}{dt}(mv)\right) = m_0 \dot{v}$

So $\underbrace{\left(m_0 + 2/3\, e^2/r_0 \cdot \frac{1}{c^2}\right)}_{m} \dot{v} = 2/3\, e^2/c^3\, \ddot{v} +$ external forces

If $m_0 = 0$ $\quad r_0 = 2/3\, e^2/mc^2$ /

Heitler [illegible] 3rd ed. discusses how in cl. theory [illegible] [illegible] field has correct [illegible] [illegible]

Either of 2 methods may be used:—

1) Subtract off an ∞ integral by separating out the part of the integrand producing the divergence in an invariant manner

1.) Evaluate the integrals using an invariant cut off λ

Then write $M = M' + M_0 + M_1\lambda + \cdots$

where $M' \to \infty$ as $\lambda \to 0$. (M' is not uniquely defined without extra prescriptions)

M' is now removed by absorbing it in the definition of the mass of the particle – write $m = m_0 + \delta m$ (i.e. $m_0 = m - \delta m$)

and express δm in terms of λ ~~the M's so as to cancel out in terms of λ~~, so that the M' for every possible diagram is exactly cancelled out.

~~So thus $M = M(m_0, \lambda) = M(m - \delta m, \lambda)$~~

M_0 is then the physically significant part of M.

This is known as mass renormalization

δm is arbitrary

choose δm so that

$M'(m-\delta m, \lambda) + M_0(m-\delta m) = M_0(m)$

stet

$M'(m-\delta m, \lambda) + M_0(m-\delta m) = M_0(m)$

$\longrightarrow \delta m = M'(m,\lambda) / \partial M'/\partial m + \frac{\partial M_0}{\partial m}$

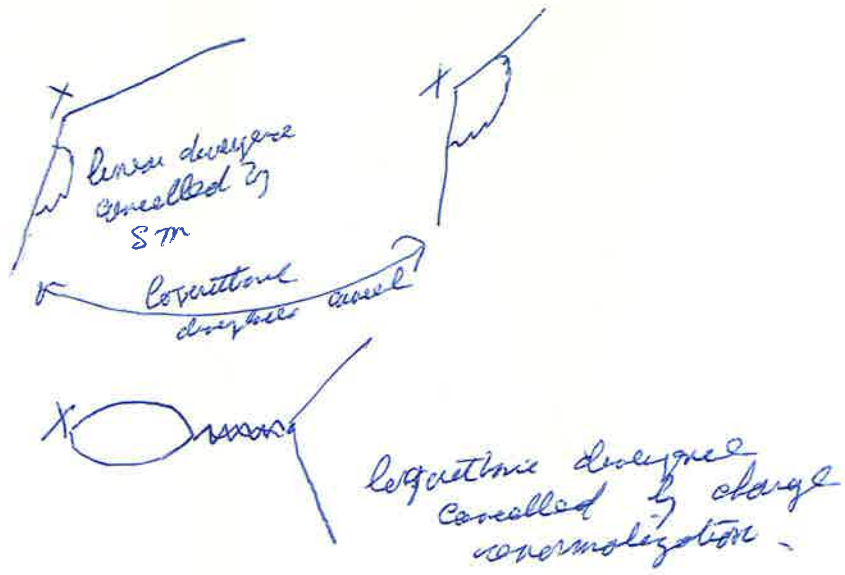
The same process can be applied to charge, write $e_1 = R(\lambda) e$ and choose $R(\lambda)$ so as to cancel logarithmic divergences which arise from the polarization of the vacuum.

Heisenberg's argument for source of divergencies

arise from conflict between relativity & q. theory. Relativity leads to sharp boundary for the region of simultaneity, in which no action can be transmitted. But this means momenta and energy must be completely undetermined, so arbitrarily high momenta and energies must occur with overwhelming probability. (H. 1958 p. 141)

ES

Coulomb scattering of an electron


linear divergence cancelled by δm
logarithmic divergence cancel
logarithmic divergence cancelled by charge renormalization.

Dyson was the first to prove that by renormalizing the mass & charge of the electron, all the divergences could be eliminated to all orders of perturbation in Q.E.D.

Meson theories spin ≥ 1 or derivative coupling for scalar or pseudo scalar mesons or Fermi interaction are non-renormalizable. Q. primitive divergence condition depends not only on no of external lines but on no of vertices (D = Const — External lines + vertices > 0) any argument of external lines can be made to diverge by taking no of vertices large enough

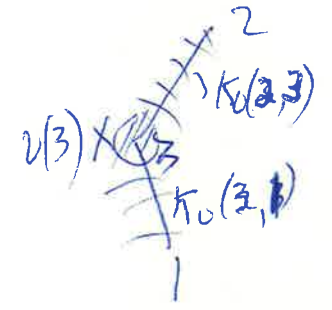
Note $\left(i\hbar\frac{\partial}{\partial t_2} - H_0\right) K_0(2,1) = \overset{i\hbar}{\hbar}\,\delta^4(x_2 - x_1)$

and $\left(i\hbar\frac{\partial}{\partial t_2} - H_0 - V\right) K(2,1) = i\hbar\,\delta^4(x_2 - x_1)$

Define $G = $ ~~[illegible]~~ $-i/\hbar\, K.$ $\quad$ $\therefore K = i\hbar G$

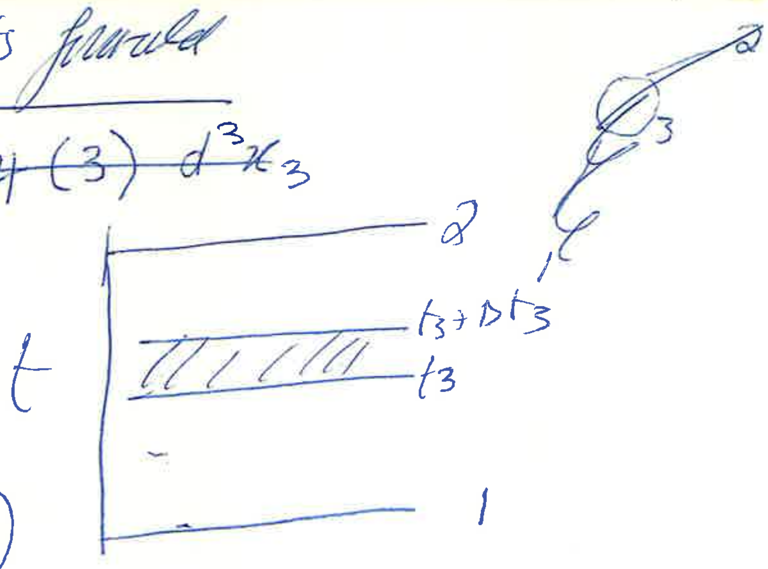
So $\left(i\hbar\frac{\partial}{\partial t} - H\right) G = \delta^4(x_2 - x_1)$

Thus in terms of diagrams



Simple proof of Feynman's formula

~~$\psi(2) = \int K_0(2,3)\,\psi(3)\,d^3x_3$~~



$$\psi(2) = \int K_0(2,3^{+\Delta t_3})\,\psi(3+\Delta t_3)\,d^3x_3$$

$$= \int K_0(2\,3^{+\Delta t_3})\left(1 - \tfrac{i}{\hbar}\left(H_0+V(3)\right)\Delta t_3\right)\psi(3)\,d^3x_3$$

$$= \iint \underbrace{K_0(2,3^{+\Delta t_3})\left(1 - \tfrac{i}{\hbar} H_0\,\Delta t_3\right)}_{K_0(2,3)} d^3x_3\; K_0(3,1)\,\psi(1)\,d^3x_1 - \tfrac{i}{\hbar}\iint K_0(2\,3)\,V(3)\,d^3x_3\,\Delta t_3\; K_0(3,1)\,\psi(1)\,d^3x_1$$

But $\int K_0(2,3)\,K_0(3,1)\,d^3x_3 = K_0(2,1)$

[since $\psi(2) = \int K_0(2,3)\,\psi(3)\,d^3x_3 = \iint K_0(2,3)\,K_0(3,1)\,\psi(1)\,d^3x_3\,d^3x_1$
$= \int K_0(2,1)\,\psi(1)\,d^3x_1$

$$\therefore K(2,1) = K_0(2,1) - \tfrac{i}{\hbar}\int K_0(2,3)\,V(3)\,d^4x_3\,K_0(3,1)$$

(now extend integration over Δt_3)

Note $K_0(2,1) = \sum_n \phi_n(x_2)\,\phi_n^*(x_1)\,e^{-iE_n(t_2-t_1)}$ $t_2 > t_1$
$= 0$ for $t_2 < t_1$

Since $\psi(2) = \sum_n e^{-iE_n(t_2-t_1)}\, C_n\,\phi_n(x \rightarrow x_2)$ ~~$\psi(x_1,t_1)$~~ But $C_n = \int \phi_n^*(x_1)\,\psi(x_1,t_1)\,d^3x_1$

$$= \int \sum_n \phi_n(x_2)\,\phi_n^*(x_1)\,e^{-iE_n(t_2-t_1)}\,\psi(1)\,d^3x_1$$

__Quote__ " $F(\epsilon^2)$ cannot be analytic & the series $F(\epsilon^2) = a_0 + a_2\epsilon^2 + \cdots$ cannot be convergent. - - - the series will be an asymptotic expansion for $F(\epsilon^2)$ in the limit of small ϵ. "

__Word 3 input__

__N.B.__ Asymptotic expansions may refer to different functions
i.e. a given expansion does __not__ characterize a function uniquely
in contrast to a power series

Near an essential singularity $F(z)$
Weierstrass's Theorem ~ near every value
Picard's theorem attains every value except at most one exceptional point

Dyson's argument re essential singularity at $e^2=0$

(1952)
P.R. 85, 631

If $\Omega \sim e^{-1/e^2} \to 0$ as $e^2 \to 0(+)$
$\to \infty$ as $e^2 \to 0(-)$

But $\Omega \sim e^{-e^2} \to$ same limit from all directions in complex e^2 plane.

Such behaviour is related to possibility of asymptotic expansions $f(z) = \Sigma a_n z^n + b_n z^{-n-1}$
$\sim \Sigma A_n z^n$ for limited region of arg z

in sense $\lim_{z\to 0} \frac{f(z) - \sum_0^n A_n(z^n)}{z^n} \to 0.$

Now for $e^2 < 0$ e^+e^- pairs would repel
Hence state of universe with real production of many pairs could be lower than vacuum (repulsion sufficient to create new pairs in effect)
Hence series blows up for $e^2 < 0$ but not for $e^2 > 0$ — ~~Asympt~~ hence essential singularity near $e^2 < 0$
"an explosive disintegration of the vacuum by spontaneous polarization" as Dyson calls it.

cf $e^{-1/w} = 1 - 1/w + \cdots$
$\to 0$ as $w \to 0$ (each term diverges as $w \to 0$.

Quote from Dyson (1949) p. 1755.

" What is to be looked for in a future theory is not so much a modification of the present theory which will make all infinite quantities finite, but rather a turning-round of the theory so that the finite quantities shall become primary and the infinite quantities secondary".

For N e^+e^- pairs with e charging

Energy $\sim -\frac{2N(N-1)}{2} \cdot \frac{e^2}{\hbar/mc} + 2Nmc^2$

$= \cancel{-} N^2 \frac{e^2}{\hbar/mc} + 2N mc^2$

< 0 if $N > 2 \cdot \frac{mc^2}{e^2/\hbar/mc} = 2\frac{\hbar c}{e^2} \simeq 2 \times 137$

All e^- in vol. $\sim \hbar/mc$
All e^+ " " $\sim \hbar/mc$ $\Big\}$ e^+e^- widely separated

Size $\hbar/mc$ because $\frac{\hbar}{\hbar/mc} \sim mc$, and $E = \sqrt{p^2c^2 + m^2c^4}$
$= \sqrt{2}\, mc^2$
Sufficient to produce pairs

Divergent integrals

[illegible] all m's & M_0

$$\int_0^\infty dk = \infty \quad \left(\int_0^L dk = L \to \infty \text{ as } L \to \infty \right)$$

$$\int_0^\infty e^{-\lambda k}\, dk = \frac{1}{\lambda} \to \infty \text{ as } \lambda \to 0$$

$$M = M'(m_0, \lambda) + M_0(m_0) + ~~\ldots O(\lambda)~~$$

$$\int_0^\infty \quad 1 + \frac{1}{\lambda}$$

$$\left(\frac{1}{\lambda} - 1\right) + 1$$

$$\left(\frac{1}{\lambda} - x\right) + x$$

$M_0(m_0\ p)$

$M_0 = 0$, when $p = 0$.

$S_e = 0$ when $p^2 = m^2$

$$\frac{1}{\lambda} + 1 + f(p) \qquad F(p) - f(m)$$

$$+ F(p) =$$

$$\frac{1}{\lambda} + 1 - f(p_0) + F(p)$$

$$L + (p^2 - m^2)\, S_e$$
$$+ (p^2 - m^2)$$

$$S_e = S_e(0) + p S_e' + p^2 S_e' + \ldots$$

$$L + \Lambda_c \qquad L + f(p)$$
$$f(p_0)$$

$$\underbrace{L + f(0)}_{M'} + \underbrace{\left(f(p) - f(0)\right)}_{M_0}$$

$$M_0(p = 0) = 0$$

* i.e. in classical limit

$$(2,1) = e^{i/\hbar\, S_c^{L_c}(2,1)}$$

ie. ~~[illegible]~~

$$(2,1) = \sum_L e^{i/\hbar\, S_c^L(2,1)}$$

S_c^L is classical action along path L.

or write $(f,i) = e^{i/\hbar\, S_c(f,i)}$ for small interval of time

then for large interval

$$(f,i) = \sum_{1,2,\cdots n} (F,1)(1,2)(2,3)\cdots(n,i)$$

→ Feynman path integral.

Action Principle in Q. Mechanics

Ref Feynman R.M.P. (1948) 20 p367.

Write $\exp\left\{\frac{i}{\hbar}\int_{t_a}^{t_b} L(t)\,dt\right\} \overset{\text{other way}}{=} B(t_b, t_a)$

$= \langle q'_{t_b} | q'_{t_a} \rangle$

$= \int\int\int \dots \langle q'_{t_b} | q'_n \rangle\, dq'_n \langle q'_n | q'_{n-1} \rangle\, dq'_{n-1}$

$\dots\, dq'_1 \langle q'_1 | q'_{t_a} \rangle$

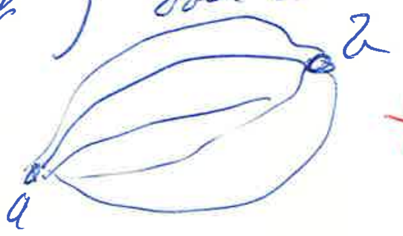
~~→ classical~~

In classical theory $B(t_b, t_a) = B(t_b, t_n)\, B(t_n, t_{n-1}) \dots B(t_1, t_a)$

Q. theory goes to classical limit at stationary phase.

classical Action function is $S = \int_{t_0}^{t} L(t)\,dt$
(Hamilton's Principal function)

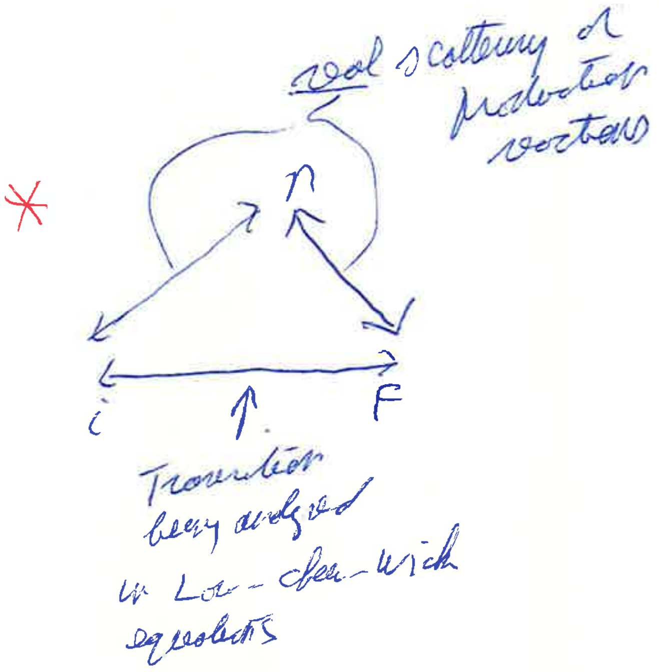
$B(t_b, t_a)$ is in general a path $\int$ over all possible paths from a to b



this is origin of Feynman's ideas

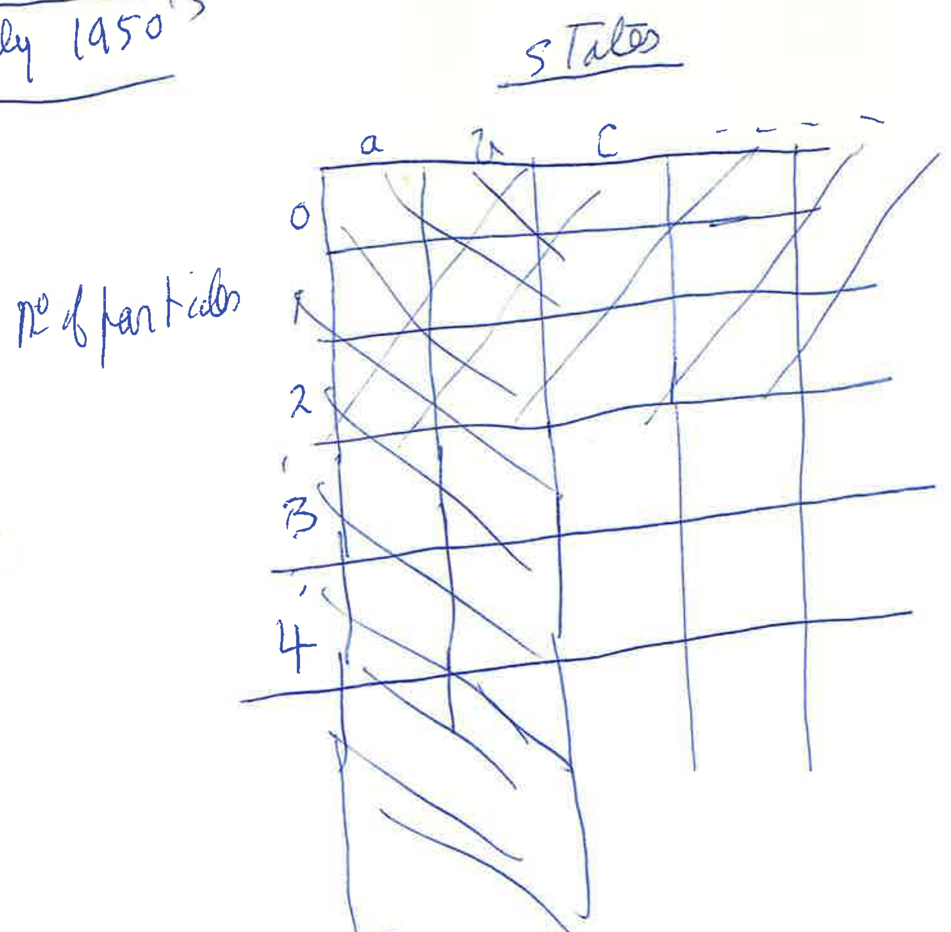
Hamilton-Jacobi Eqn is $\frac{\partial S}{\partial t_0} = H\left(q_0, -\frac{\partial S}{\partial q_0}, t_0\right)$ (element q_0 in place of q)

$p_0 = -\partial S / \partial q_0$, $S = S(q, q_0, t, t_0)$


real scattering of radiation vectors
n
i
F
Transition being analysed in Low-Chew-Wick equations

Approximation Schemes

Early 1950's



Tamm-Dancoff

a few particles in all states

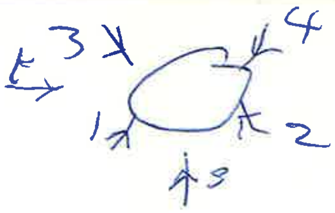
Tomonaga

all numbers of particles in a few states

Low-Chew-Wick   non-linear eqns connecting transition amplitudes to amplitudes for all processes beginning with the initial state & ending on the final state* — now neglect some possible processes cp. with process with in which amplitude's for various virtual states of the field are linked & some of those amplitudes are neglected → analytic properties → S-matrix theory.

Crossing symmetry

(18)



I. s – channel $\quad 1+2=3+4$

II. t – channel $\quad 1+\bar{3}=\bar{2}+4$

III. u – channel $\quad 1+\bar{4}=3+\bar{2}$

PCT $\bar{3}+\bar{4}=\bar{1}+\bar{2}$

$s=(p_1+p_2)^2$, $\quad t=(p_1+p_3)^2$

$u=(p_1+p_4)^2$

for t-channel $\quad p_1'=p_1 \quad p_3'=p_2$

$p_2'=p_3 \quad p_4'=p_4$

so $s'=t$, $\quad t'=s$, $\quad u'=u$

u channel $\quad p_1'=p_1 \quad p_3'=p_3$

$p_2'=p_4 \quad p_4'=p_2$

so $s'=u$, $\quad t'=t$, $\quad u'=s$

P.C.T.

$p_1'=p_3$

$p_2'=p_4$

$p_3'=p_1$

$p_4'=p_2$

$s'=s,\ t'=t,\ u'=u$

$F_{PCT}(s',t')$

$=F_I(s',t')$

$\therefore F_{PCT}(s,t) = F_I(s,t)$

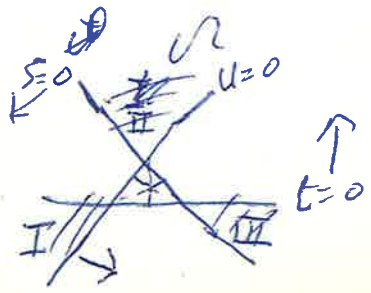
then $F_{II}(s',t') = F_I(s,t) = F_I(t',s')$

or $F_{II}(s,t) = F_I(t,s)$

and $F_{III}(s',t') = F_I(u',t')$

$= F_I(4-s'-t',t')$

$F_{III}(s,t) = F_I(4-s-t,t)$



$s+t+u$ or area Δ = const.

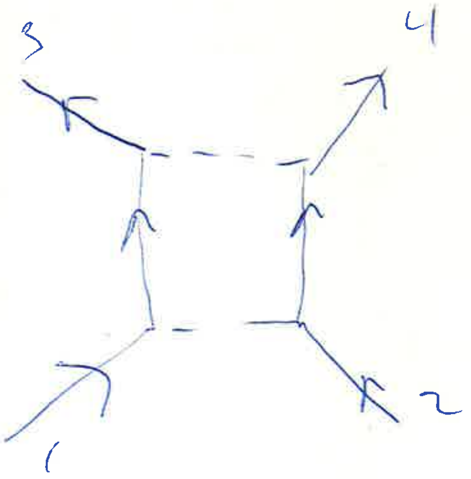
also → $6+2p_1\cdot p_2+2p_1\cdot p_3+2p_1\cdot p_4$

but ~~$(p_1+p_2+p_3+p_4)$~~ $p_1=-p_2-p_3-p_4$

$p_1^2=1=-p_1\cdot p_2-p_1\cdot p_3-p_1\cdot p_4$

or $s+t+u=4$

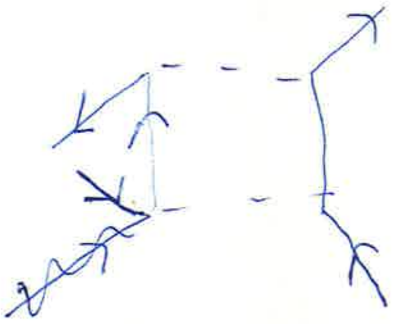
$\underline{\Omega}$



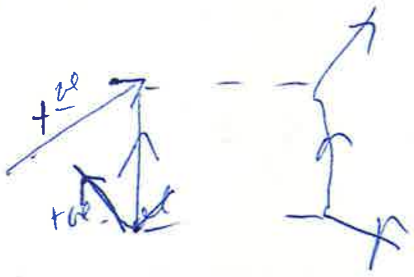
$1+2 \to 3+4$

cross. 1 and 3 we get $2+\bar{3} \to \bar{1} + 4$

we get



we interpret this as



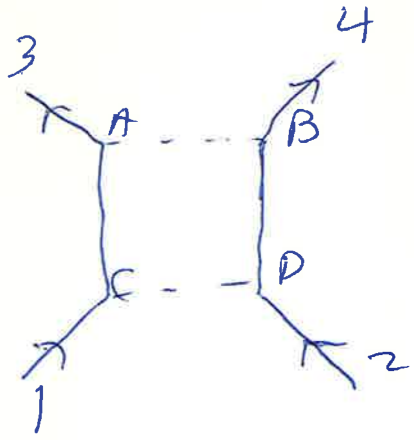
So $p_1 \to$ [illegible] $+p_3$

$p_1 \to$

So $p_1 \to p_1' = -p_3$
$p_3 \to p_3' = -p_1$

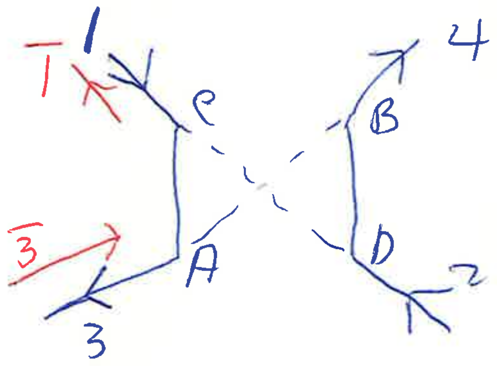
Crossing

Consider Feynman diagram



Cross 3 & 1 means substitute $p_3 \rightarrow p_3' = -p_1$

$p_1 \rightarrow p_1' = -p_3$



then $F(2+\bar{3} \rightarrow \bar{1} + 4)$

is derived from $F(1+2 \rightarrow 3+4)$

by substituting $\left.\begin{matrix} p_3 \rightarrow -p_1 \\ p_1 \rightarrow -p_3 \end{matrix}\right\}$

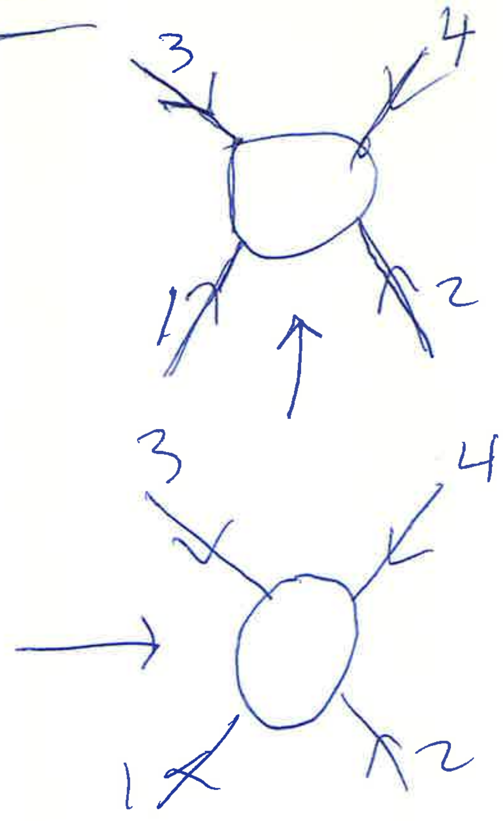
Suppose we cross 2 & 3

new reaction is $1+\bar{3} = \bar{2}+4$

This can be viewed by leaving the lines uncrossed and changing our line of sight along which the

reaction process

Has



$1+2=3+4$

$1+\bar{3}=\bar{2}+4$

Define $s=(p_1+p_2)^2$, $t=(p_1+p_3)^2$. $u=(p_1+p_4)^2$

$p_1+p_2+p_3+p_4=0$

$$s+t+u = 6 + 2p_1\cdot p_2 + 2p_1\cdot p_3 + 2p_1\cdot p_4$$

$\hookrightarrow -2p_1\cdot(p_1+p_2+$

$\longrightarrow 4.$

~~In~~ In s-channel

$s = 4E_s^2$
$t = -4p_s^2 \sin^2 1/2\,\theta_s$
$u = -4p_s^2 \cos^2 1/2\,\theta_s$

t-channel

$s = -4p_t^2 \sin^2\frac{1}{2}\theta_t$
$t = 4E_t^2$
$u = -4p_t^2 \cos^2\frac{1}{2}\theta_t$

u-channel

$s = -4p_u^2 \cos^2\frac{1}{2}\theta_u$
$t = -4p_u^2 \sin^2\frac{1}{2}\theta_u$
$u = 4E_u^2$

and $F_t(s,t) = F_s(t,s)$

is another way of expressing all this.

thus Take $F(s, t, u)$

and substitute the roles for s t , u

and these will lead to the amplitudes

F_s , F_t , F_u in different parts of the stu plane

e.g. $F_t(S,t) = F_s(t,S)$

If ~~[illegible]~~ s , t channel identical

then $F(s,t) = F(t,s)$

is crossing symmetry for the amplitude between the s , t channels.

The analytic S-matrix

⑩ 1

Definition of S-matrix S-operator $\hat{S} = U(\infty, -\infty)$

= time evolution operator between initial & final scattering states

If $|\psi_i\rangle$ is initial state at $-\infty$

state at $+\infty$ is $\hat{S}\,|\psi_i\rangle$

$$= \sum_f S_{fi}\;|\psi_f\rangle$$

$$\text{where } S_{fi} = \langle \psi_f|\,\hat{S}\,|\psi_i\rangle$$

= S-matrix element for scattering from i to f.

probability of scattering $\propto |S_{fi}|^2$

Unitarity $P_f = |S_{fi}|^2$

But $\sum_f P_f = 1 = \sum_f S_{fi}\,S_{fi}^{*} = \sum_f S^{+}_{if}\,S_{fi}$

$$= (S^{+}S)_{ii} = 1$$

$$\underline{S^{+}S = 1}$$

(for non diagonal elements choose $|i\rangle = (\alpha|a\rangle + \beta|b\rangle)(|\alpha|^2+|\beta|^2)^{-1/2}$.
for arbitrary α & β $\rightarrow$ $\langle a|S^{+}S|b\rangle = 0$)

Schwarz Reflection Principle

$f(\overline{z}^{*}) = \overline{f(z)}$

if $f(z)$ is real on real axis.

Write $S = 1 + iR$.

then $(1 - iR^{\dagger})(1 + iR) = 1$

or $i(R - R^{\dagger}) = -R^{\dagger}R$

$$i\left(R_{fi} - R^{\dagger}_{fi}\right) = -\sum_q R^{\dagger}_{fq} R_{qi}$$

But $R^{\dagger}_{fi} = R^{*}_{if} = R_{fi}^{*}$ under PT invariance

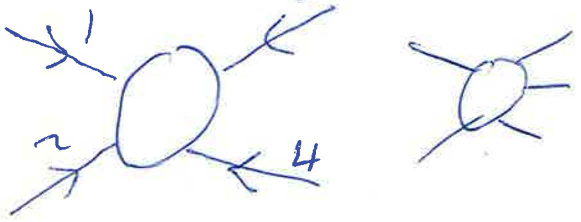
$$\therefore\; 2\,\mathrm{Im}\,R_{fi} = \sum_q R^{\dagger}_{fq} R_{qi}$$

$$= \sum_q R^{*}_{qf} R_{qi} \quad \boxed{= \sum_q R_{fq}^{*} R_{qc}}$$

[In general L.H.S. ∝ disc. across cut — function analyticity

as new q's are permitted $\mathrm{Im}\,R_{fi}$ picks up new terms

ex In 2-body scattering particles mass m



$s = (p_1 + p_2)^2 = 4E^2 > 4m^2$

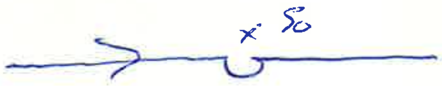
$t = (p_1 + p_3^{2})^2 = -4p^2 \sin^2 \tfrac{1}{2}\theta < 0$

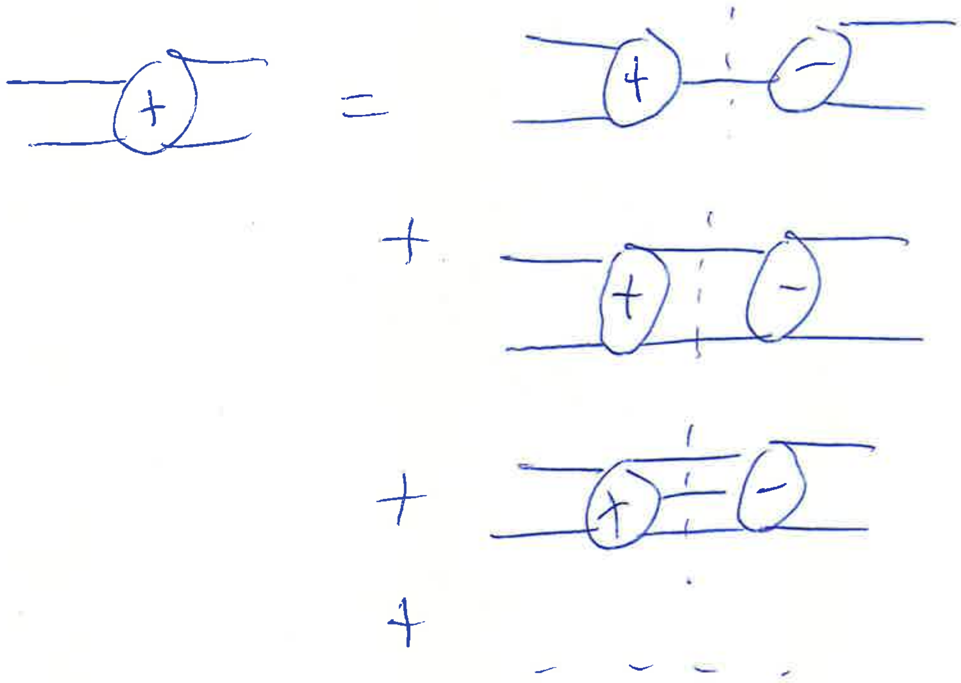
$u = (p_1 + p_4)^2 = -4p^2 \cos^2 \tfrac{\theta}{2} < 0$

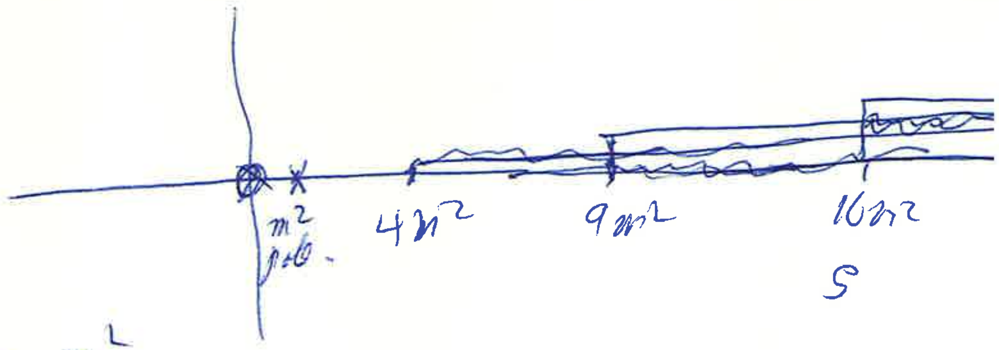
$s + t + u = 4(E^2 - p^2) = 4m^2$

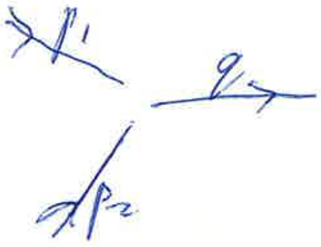
threshold for 2 body scattering is $s = 4m^2$
— — . production 1 particle $9m^2$
2 . . $16m^2$

__N.B.__ $\frac{1}{s-s_0} = P\left(\frac{1}{s-s_0}\right) + i\pi\,\delta(s-s_0)$ LS



Im






Notice pole at $s = m^2$
when $s = (p_1 + p_2)^2 = q^2 = m^2$ thus
But this is below threshold
since $s \geq 4m^2$, so
conservation laws do not allow
the process in the physical region
but is essential when we move out into
the whole region of the s-plane.

S-matrix & bound-states in N.R. scattering Theo

ψ satisfies S. eq. ~~$-\frac{\hbar^2}{2m}\nabla^2 + V$~~

$$\left(\nabla^2 + 2m/\hbar^2 (E - V)\right)\psi = 0.$$

$\psi \sim e^{ikz} + r^{-1} e^{ikr} f(\theta)$ $\quad E = \frac{\hbar^2 k^2}{2m}$, $U = 2m/\hbar^2 \cdot V$

$\sigma \propto |f(\theta)|^2$ $\quad$ So Eq is $\nabla^2\psi + (k^2 - U)\psi = 0$

notice $e^{ikz} = \sum_{\ell=0}^{\infty} (2\ell+1) i^\ell j_\ell(kr) P_\ell(\cos\theta)$, $j_\ell(x) \sim \frac{1}{x}\sin($

$\sim i/2kr \sum_\ell (2\ell+1)\left[(-1)^\ell e^{-ikr} - e^{ikr}\right] P_\ell(\cos$

Asymptotic soln is well-known.

$$\psi \sim \frac{1}{kr}\sum_{l=0}^{\infty}(2l+1)\, i^{l} e^{i\eta_l}\, P_l(\cos\theta)\, \sin\left(kr-\tfrac{1}{2}l\pi+\eta_l\right),$$

where η_l is phase-shift induced by potential.

$$f(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left(e^{2i\eta_l}-1\right) P_l(\cos\theta)$$

$$= \sum (2l+1)\, a_l(k)\, P_l(\cos\theta)$$

$$a_l(k) = \frac{e^{2i\eta_l}-1}{2ik} = \frac{e^{i\eta_l}\sin\eta_l}{k} = \frac{1}{k(\cot\eta_l - i)}$$

Notice $\psi \sim \frac{i}{2kr}\sum_l (2l+1)\left[(-1)^l e^{-ikr} - e^{2i\eta_l} e^{ikr}\right] P_l(\cos$

We define $S_l = e^{2i\eta_l}$ = partial wave S-matrix element

So $\psi_l \sim \left[(-1)^l e^{-ikr} - S_l e^{ikr}\right] P_l(\cos\theta)$

Suppose $S_l \to \infty$, then $\psi \sim e^{ikr}$

if $k = i\kappa$ when $S \to \infty$, $\psi \sim e^{-\kappa r}$, bound-state

But $E \propto k^2$ ∴ +ve imaginary axis for k is real $E < 0$.

cut E-plane is a 2-sheeted Riemann surface. upper $\frac{1}{2}$ of k-plane → 1st sheet; lower $\frac{1}{2}$ of – – → 2nd sheet $\Big\}$ E-

So bound-states are related to poles on the −ve E-axis similarly poles in the complex E-plane on the 2nd sheet correspond to resonances — In the k-plane $k = a - ib$

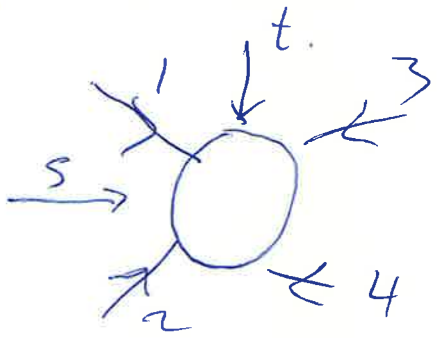
gives time dependence $e^{-iEt} \to e^{-2abt}$ × harmonic term $a > 0,\ b > 0$

(space dependence ∝ e^{bR} is correct for a decaying state) = decaying state.

poles at $-ib$, $b > 0$ i.e. −ve imaginary axis give asymptotic behaviour e^{bR} correspond to virtual states

(Note resonances, virtual states occur on the unphysical sheet in the E-plane.)

crossing symmetry



amplitude for $1 + 2 = 3 + 4$, so called s-channel.

also gives amplitude for

$1 + \bar{3} = \bar{2} + 4$, t channel

and $1 + \bar{4} = 3 + \bar{2}$, u-channel

t-channel

$t = 4k_t^2$
$s = -4k_t^2 \cos^2 \frac{1}{2}\theta_t$
$u = -4k_t^2 \sin^2 \frac{1}{2}\theta_t$

u-channel

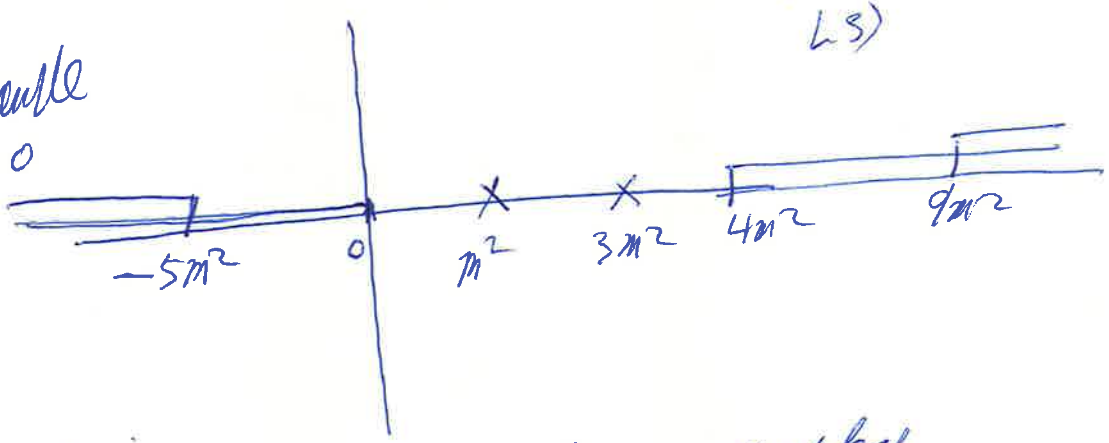
$u = 4k_u^2$
$s = -4k_u^2 \sin^2 \frac{1}{2}\theta_u$
$t = -4k_u^2 \cos^2 \frac{1}{2}\theta_u$

Crossing symmetry gives ~~cuts~~ branch points
for fixed t
at $u = 4m^2, 9m^2 \ldots$ pole at $u = m^2$

this implies $s = 4m^2 - u - t = -t, -t - 5m^2 \ldots$ branch pts

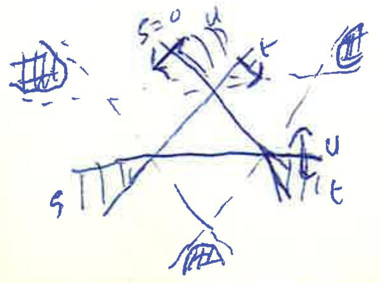
$= 3m^2 - t$ for pole
(s)

Consider for example
the case $t = 0$



note t is physical when t becomes complex
poles etc move onto the complex s plane but
remain on real axes in product of $s\,t > u$ planes

Mandelstam representation $F(s,t,u) = \text{Pole terms} + \frac{1}{\pi^2}\iint_{4m^2}^{\infty} \frac{\rho_{st}(s',t')\,ds'dt'}{(s'-s)(t'-t)}$
$+ \ldots$



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t- channel

$t = 4E_t^2 \quad > 4m^2$

$s = -4p_t^2 \sin^2\frac{1}{2}\theta_t < 0$

$u = -4p_t^2 \cos^2\frac{1}{2}\theta_t . < 0$

i.e. $\left.\begin{matrix} t \to s \\ s \to t \end{matrix}\right\}$

u- channel

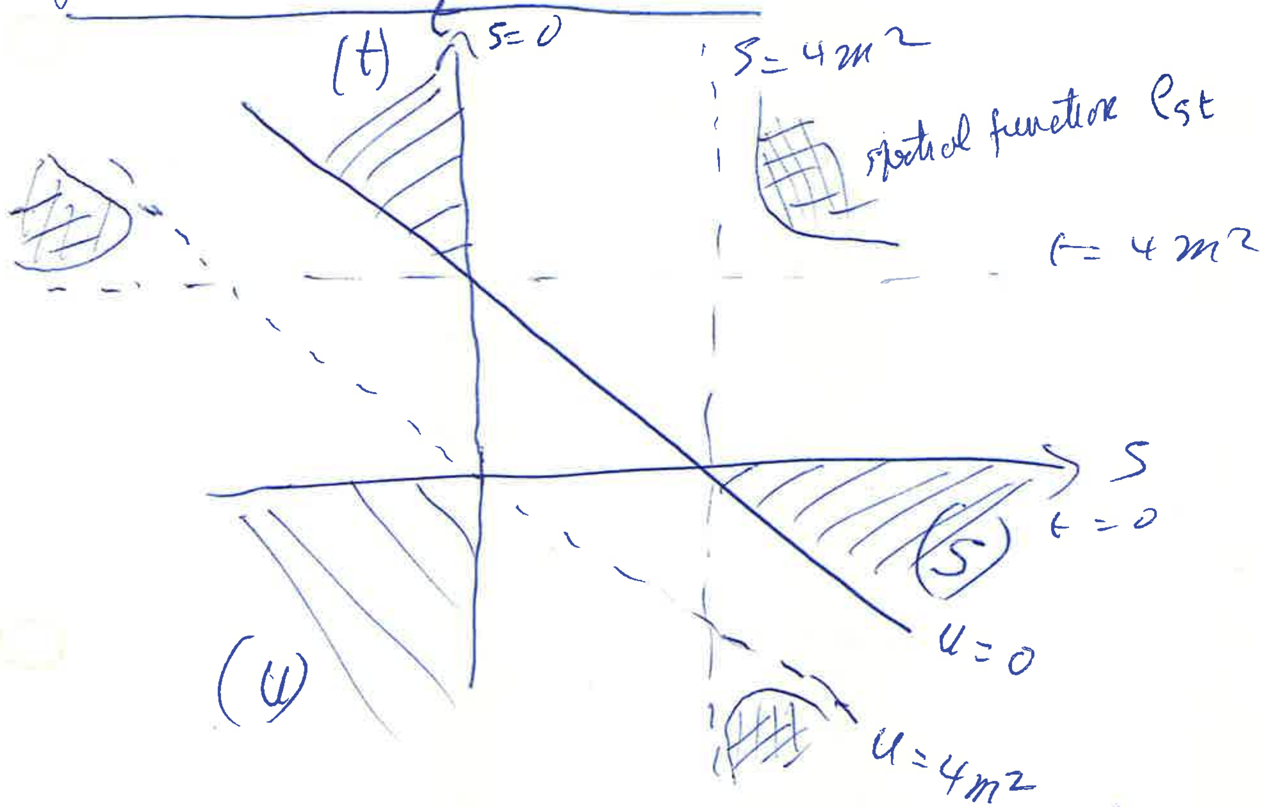
$u = 4E_u^2 \quad > 4m$

$s = -4p_u^2 \cos^2\frac{1}{2}\theta_u . < 0$

$t = -4p_u^2 \sin^2\frac{1}{2}\theta_u < 0$

i.e. $\left.\begin{matrix} u \to s \\ s \to u \end{matrix}\right\}$

The Mandelstam diagram


(t)
t
s=0
s=4m²
spectral function ρ_st
t=4m²
S
t=0
(S)
u=0
(u)
u=4m²

Ed

(s) ~~$\bar{\pi} + N = \bar{\pi} + N$~~

$N + \bar{\pi} = N + \pi$

(t) ~~$\bar{\pi} + \pi = N + \bar{N}$~~

$N + \bar{N} = \pi + \pi$

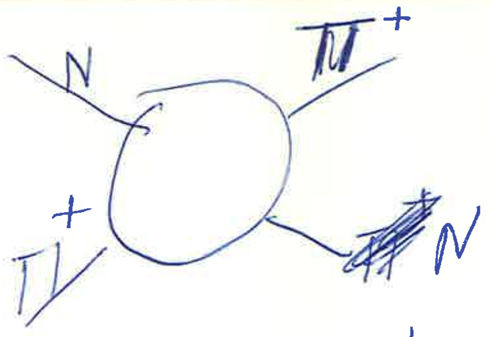
(u) $\bar{N} + \pi = N + \pi$

Why we need the Mandelstam representation

Note The introduction of the Mandelstam representation is due essentially to the fact that $F(s,t)$ is not determined, but merely constrained, by the single variable dispersion relations. To find $F(s,t)$ we must know singularities in t as well as in s, u. By introducing arbitrary singularities in t we could clearly alter $F(s,t)$ without altering the fixed t dispersion relation, which does not involve singularities in t in its formulation. Hence the single variable disp. rels do not determine F as a function of s, t uniquely — We must go to double dispersion relations & work in terms of ∫al Eq for double spectral function determined from unitarity (this is Mandelstam's own approach) or replace $F(s,t)$ by the set $\{a_l(s)\}$ of partial wave amplitudes for which complete dispersion rels can be written down by using singularity structure in s, u and t (i.e. eg. Mandelstam representation)

Essentially both u & t singularities produce "forces" or potentials in the s-channel, and the single variable relations do not take account of the t-channel forces (cf Chew (1959) p 2)

Ex



3 different process described by one amplitude $F(s, t)$

(s) $N + \pi^+ =$ $\pi^+ + N$ ~~$N + \pi^+$~~

~~(t) $N + \bar{N} = \pi^- + \pi^+$~~

(t) $N + \pi^- =$ $\pi^- + N$ ~~$N + \pi^-$~~

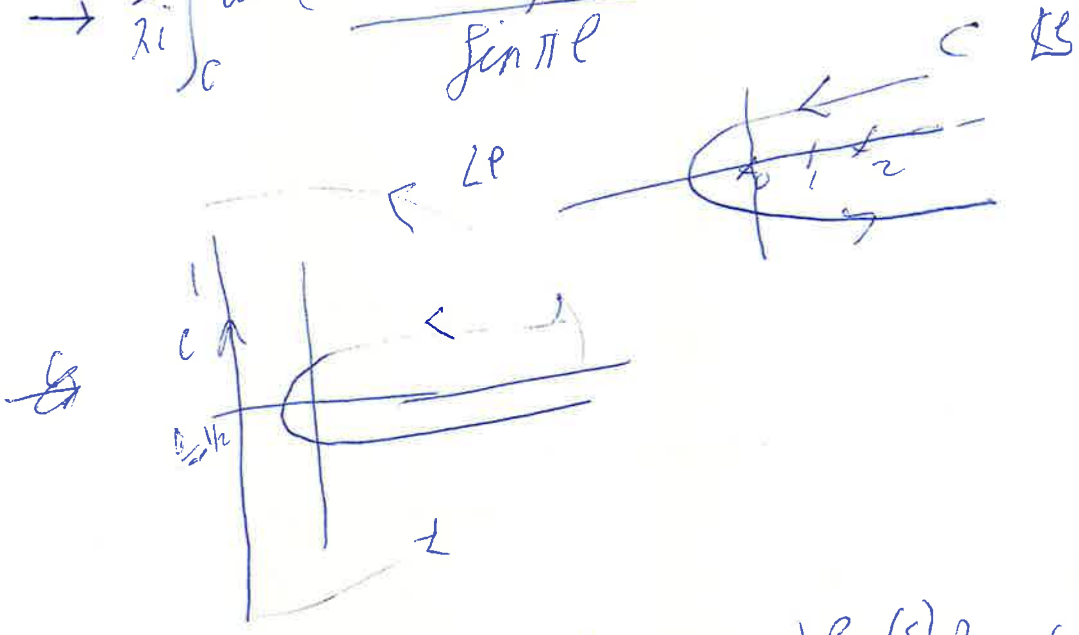
(u) $N + \bar{N} = \pi^+ + \pi^-$

TCP inverses of s-channel is $\bar{3} + \bar{4} = \bar{1} + \bar{2}$

similarly for t, u-channel.

also PT invariance brings in the inverse reactions, so we may consider 12 different processes all together then $F(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, a_\ell(s)\, P_\ell(\cos\theta)$

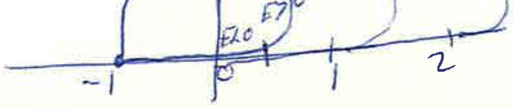
$$\rightarrow \frac{1}{2i}\int_C d\ell\, \frac{(2\ell+1)\, a(\ell,s)\, P_\ell(-\cos\theta)}{\sin \pi\ell}$$



$$\rightarrow -\frac{1}{2i}\int_{C'} \;-\; \pi \sum_n \frac{(2\alpha_n(s)+1)\,\beta_n(s)}{\sin \pi\alpha_n(s)}\, P_{\alpha_n(s)}(-\cos\theta)$$

where $a(\ell,s) \simeq \frac{\beta_n(s)}{\ell-\alpha_n(s)} + \cdots$

For Yukawa potential trajectory is this:—

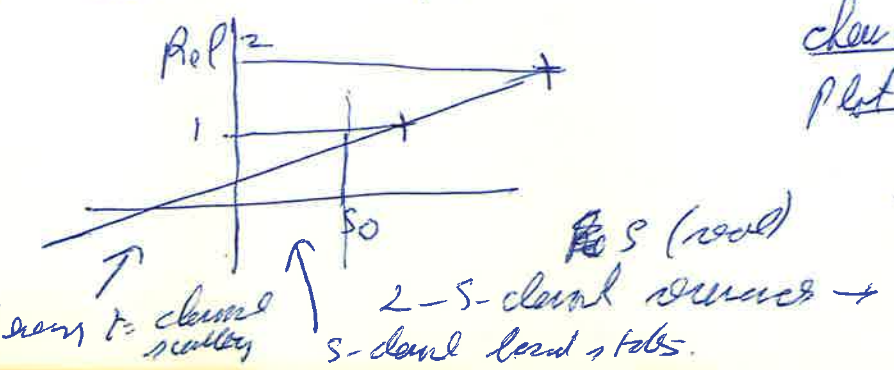


→ asymptotic form $s^{\alpha(t)}$ in the s channel in terms of t-channel Regge poles

Each channel brings in its own set of singularities, cuts, poles by unitarity. Thus by considering crossing we obtain all the necessary singularities in the amplitude $F(s,t)$.

Complex Angular momentum

~~Instead~~ we can expand $F(s,t)$ in terms of partial wave amplitudes $a_\ell(s)$ now consider ℓ as a complex variable $a_\ell(s) \to a(\ell, s)$ poles in ℓ for given s are called Regge poles — as s varies they move about the ℓ plane. Values of s on the real axis for which poles in ℓ occur at integral values for $\text{Re}\,\ell$ give the bound states, resonances (for bound states $\text{Im}\,\ell = 0$ at pole)

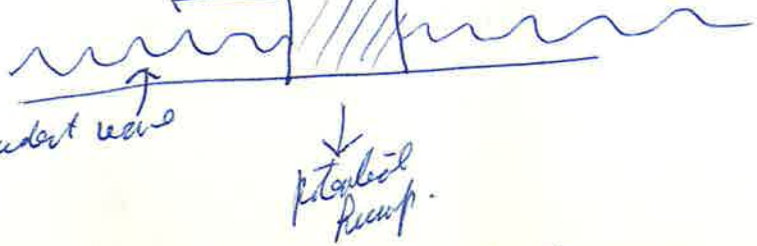


Chew-Frautschi plot of a Regge Trajectory

s low diag.
p. 455
Martin & Spearman

We may distinguish the Mandelstam conjecture that F(s, t) is an analytic function of s & t except at singularities generated by unitarity (including anomalous thresholds etc which are necessitated by normal threshold structure when the unitarity eqn is "iterated") from the Mandelstam representation which a double dispersion relation, not valid for anomalous thresholds, but has proved a useful heuristic device — proved in potential theory by Regge — Regard [illegible] for his work.

* Mesons could be dis:—



what [illegible] at $z=0 = B(t)$
called . . . $= A(t)$

$B(t) = \int a(\omega)\, e^{i\omega t} = 0, \; t<0$

$A(t) = \int g(\omega)\, a(\omega)\, e^{i\omega t} = 0 \; t<0$

where $a(\omega)$, $g(\omega)a(\omega)$ are analytic functions of ω

alternatively at $l=$ integer ($\mathrm{Im}\, l = 0$) poles occur near real s-axis. In any event $\mathrm{Re}\, s \approx \mathrm{Re}\, l =$ integer are near the position of a pole.

Analyticity

So far we have discussed the singularities of the S-matrix required by unitarity, crossing and particles. How does it behave away from these singularities?

We assume with Mandelstam that everywhere else ~~the~~ $F(s,t)$ is an analytic function of s and t.

This is loosely related to causality.

Eg $vt - r < 0$ implies $A(r,t) = 0$ for a scattering amplitude

~~write $r = vt + r$~~ (~~eg~~: $A(0,t)=0$ for $t<0$)

$A(t) = 0$ for $t < 0$.

Consider Fourier transform $~~g(t)~~ g(\omega) = \int_{-\infty}^{\infty} A(t) e^{-i\omega t} \, dt$

then $g(\omega)$ is an analytic function of ω in the lower half plane. (measures scattering amplitude at energy $\hbar\omega$)

But such arguments are merely heuristic

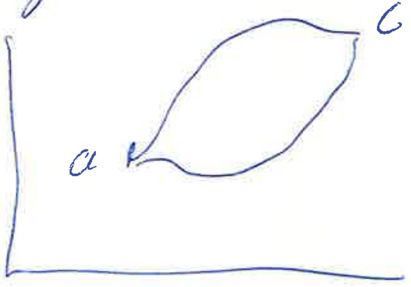
Analyticity is a new postulate

Broadly the same situation applies when F is considered as a function of $s\ t$ & u.

$$F(s,t,u) \xleftarrow[\text{Mandel. rep.}]{} \underset{\text{(residues, discontinuities \& poles)}}{\text{singularities}} \xleftarrow[\text{unitarity \& crossing}]{} \text{amplitudes for reactions direct \& crossed channels}$$

$F(s,t,u) \to F_{I}, F_{II}, F_{III}$ by choosing appropriate values for $s\ t$ & u.

For an analytic function $\int_a^b f(z)\,dz$ does not depend on path from a to b if no singularity is trapped between the paths.

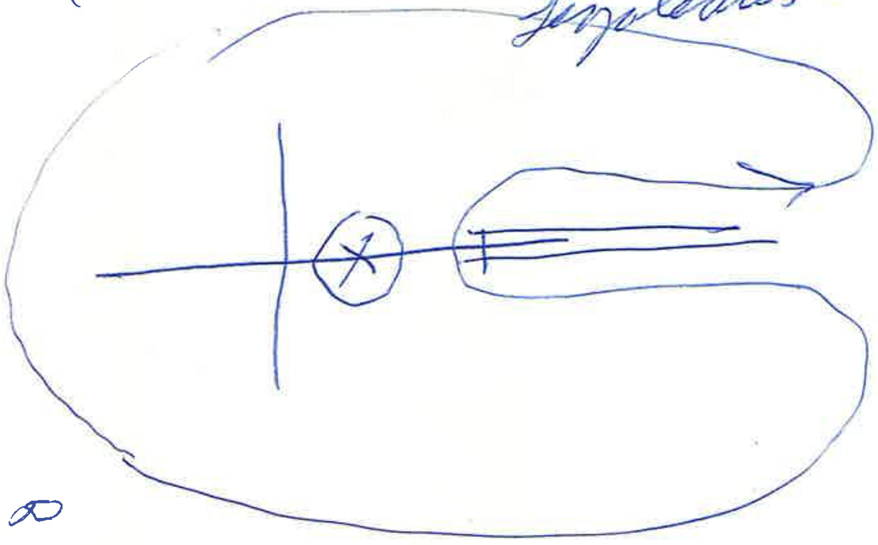


By Cauchy's theorem

$$f(z) = \frac{1}{2\pi i} \oint_C \frac{f(z')}{z'-z}\,dz'$$

C encloses no singularities.

eg.



with suitable behaviour at ∞

Analyticity: $F(z) \underset{\text{determined by}}{\uparrow}$ singularities (residues, discontinuities & their positions)

Unitarity singularities ↑ all relevant amplitudes including crossed channels.

So $F(z) \underset{\underline{\text{analyticity}}}{\longleftrightarrow}$ singularities $\underset{\underline{\text{unitarity \& crossing}}}{\longleftrightarrow} F_1(z), F_2(z) \ldots$

leads to coupled eqns for the amplitudes.

α cutoff due to high energy behaviour
for high spins like s^j
this can be controlled by assuming
delicate cancellations due to families
of particles on Regge trajectories
amplitude ~ $s^{\alpha(t)} \to$ < [illegible] for t < 0
some disagreement as whether cutoffs are
intrinsic to theory: Jaroo says yes
Chew says no

But cutoffs are used in practice even in strip approx.
cf Alber & Zomach
Fulco, Frazer & Wong } for N/D

historical note Gell mann (1956) was first to suggest
using dispersion relations to replace field eqs (cp Mandelstam 1958)

Resulting Equations may have. no solution, with given singularities (particle poles), but do have solns with new poles

If these are now introduced into singularity structure in general we get new equations to solve again.

are there a self-consistent set of poles, such that amplitudes are uniquely determined

Possibilities: (1) more than one set of poles is possible

(2) the system is over constrained, so no set of poles exist

(3) there is a unique set of poles which yield self consistency: this is the Bootstrap hypothesis of Chew > Frautschi

But set may depend on remaining cutoff parameters arising in coupling of these ≥ 1 as in non-renormalizable meson theories (Gasiorowicz p. 430)] *

1

Regge theory and maximal analyticity of the second kind.

We can express $F(s,t)$ analyticity in terms of analyticity properties of partial waves i.e. $\{a_l(s)\}$. L.H. cut in $a_l(s)$ comes from singularities in crossed channel for all values of l i.e. crossing "mixes up" partial waves

N/D soln of partial wave dispersion relations expresses $a_l(s) = \frac{N_l(s)}{D_l(s)}$

C.D.D. ambiguity ~~is~~ exists

$$D_l \to D_l + \sum_{i=1}^{P_l} \frac{\alpha_i}{s-s_i}$$

α_i, s_i are arbitrary parameters.

These poles in D_l are only allowed for $l = 0$ or 1 (See below for reason).

Poles in D_l are equivalent to zeros in D_l which correspond to fixed J singularities in a_l — these are the elementary-particle poles, (sometimes also referred to as C.D.D. poles)

Why fixed J-planes not allowed for $J > 1$

(i.e. elementary particles (arbitrary mass coupling) are only possible for $J = 0\ \frac{1}{2}$ or 1.)

This is because the presence of such poles would break the Froissart bound

Froissart says.

$$\text{Im}\, F(s, t=0) < \text{const.}\ s (\ln s)^2.$$

$$\hookrightarrow \propto (E\, p)\, \sigma_{tot}.$$

This because. $F(s,t) = E\, f(\theta)$ $\hookrightarrow$ scattering amplitude $\frac{d\sigma}{d\Omega} \propto |f(\theta)|^2$.

and by optical theorem

$$\text{Im}\, F(\theta = 0) \propto p \cdot \sigma_{tot}.$$

$$\therefore\ \sigma_{tot} < \text{const.}\ \frac{s}{Ep} (\ln s)^2 \sim (\ln s)^2.$$

This is derived by following argument.

Interaction potential $V \sim g(s)\, e^{-\alpha r}$ if $g(s) \sim s^n$.

$$= e^{-(\alpha r - n \ln s)}$$

defines an effective radius R such that

$\alpha R - x \ln s = k \gg 0$

i.e. $R \simeq k + \text{const}\cdot\ln s$

a R increases as $\ln s$ effectively

$\sigma_{tot} \lesssim \pi R^2 \propto \ln^2 s$, originally derived from unitarity eqn and special restrictions imposed by analyticity on energy dependence of effective interactions

In fact Froissart was able to prove the stronger result

$$|F(s,t=0)| < c\, s \ln^2 s$$

---

now consider a pole of angular momentum J, mass M in the $t=$ channel.

then will calculate

$$T(s, t \simeq M^2) \simeq \frac{g^2 P_J(\cos\theta_t)}{M^2 - t}$$

now we have in $t=$ channel.

$$t = 4E_t^2 = 4(p_t^2 + m^2)$$

$$s = -2p_t^2(1-\cos\theta_t) \quad \therefore \cos\theta_t = \left(1 + \frac{2s}{t-4m^2}\right)$$

We can now continue to large +ve s
and use $P_J(\mu) \sim \mu^J$
to give a contribution involving a factor
$$P_J\left(1+\frac{2s}{t-4m^2}\right) \sim s^J$$

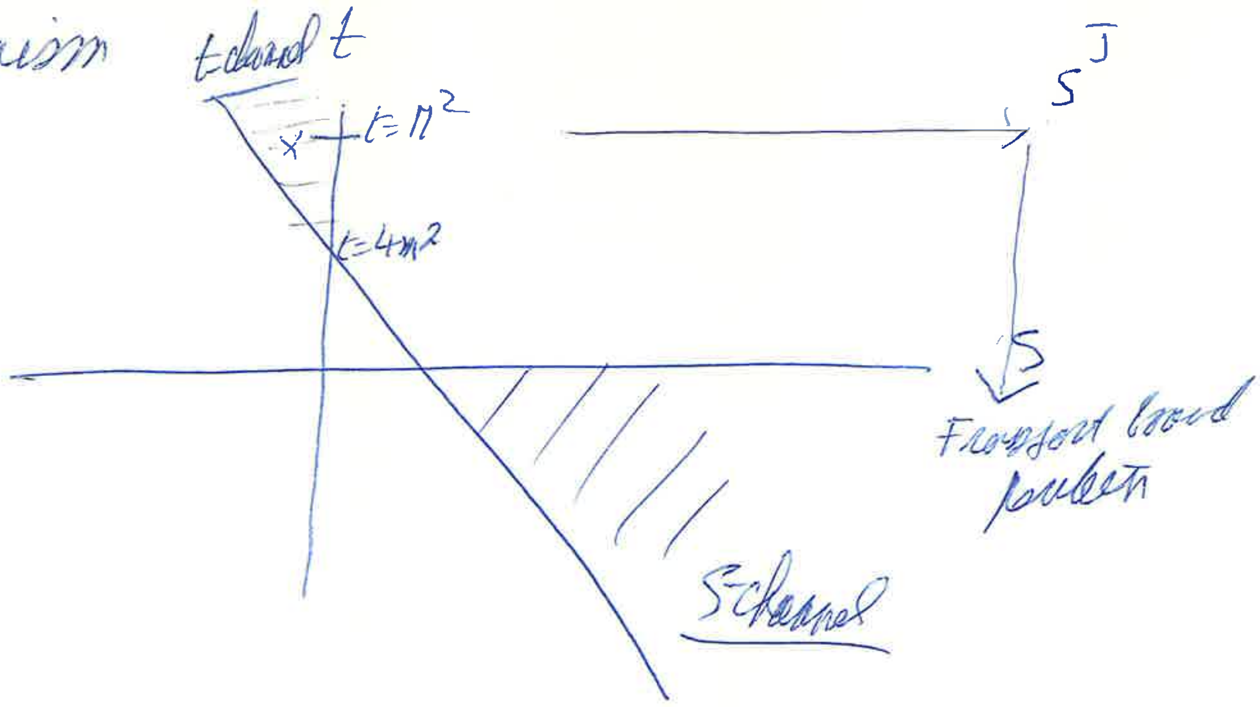
If J is fixed, does not depend on t
we can continue down to $t \leq 0$ (from $t = M^2$)
to obtain a contribution to the $F(s, t=0)$
which behaves like s^J and thus
will break the Froissart bound
for $J > 1$. Hence the arbitrary
C.D.D. poles are restricted to $J \leq 1$.

But suppose J, the pole position, is a function
of t and $J(t=0) < 1$, although $J(t=M^2)$
= integer ≥ 1
then we could accommodate particles
with spin greater than 1 in the
t-channel. This is essentially the Regge.

*
Note Froissart-Gribov expresses $a(l, s)$ in terms of discontinuities in the t channels which will depend on C.D.D. poles for low l in crossed channels, but it does not itself introduce extra ambiguity in the direct (s) channel. In terms of N subtractions, for $l > N$, the subtraction polynomial does not contribute to Froissart Gribov.

So cycle is thus:— C.D.D. poles for low l in crossed channels determine high l waves in s-channel (cond. of 1st kind) — continuation in l in s-channel then determines low l poles in s-channel (cond. of 2nd kind) and we require self consistency between input and output poles.

Mechanism


t-channel t
$t = \mu^2$
$t = 4m^2$
S^J
S
Froissart bound problem
S-channel

To eliminate the existing C.D.D. poles for $J \leq 1$, we proceed as follows.

first interpolate $a_\ell(s) \to a(\ell, s)$ *
by the Froissart-Gribov representation
this enables a continuation to be made
into the complex ℓ-plane for large ℓ.
There is no necessity that if this function
is continued in ℓ to low ℓ-values
we will obtain the physical amplitude
for $\ell = 0$ or 1, since there are essentially

Kay Small

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not defined by analyticity requirements so far imposed (maximum analyticity of the 1st kind). (i.e. C.D.D. ambiguity exists).

Now assume principle of maximal analyticity of the 2nd kind — that $a_\ell(s)$ for low ℓ (i.e. = all ℓ) are obtained by interpolation in ℓ from the high values.

This is equivalent to saying $a(\ell, s)$ is analytic in ℓ as well as s apart from poles ~~poles~~ or branch points. (Regge poles & cuts)

This has been shown by Mandelstam (1963) to be equivalent to setting the $\lambda_i = 0$ in the C.D.D. ambiguity.

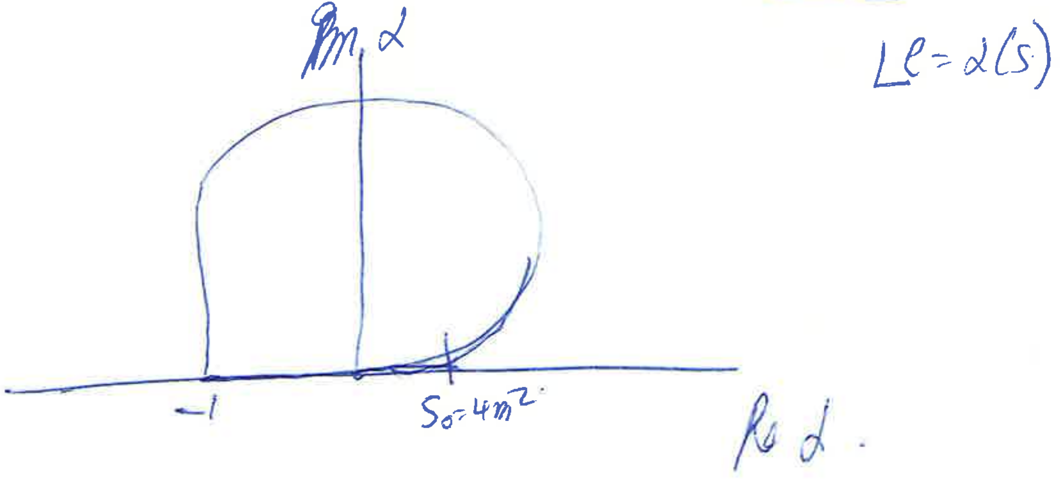
Using the interpolation function $a(\ell, s)$ we can now proceed to a representation of $F(s,t)$ via the ~~Watson - Sommerfeld~~ Sommerfeld - Watson transform.

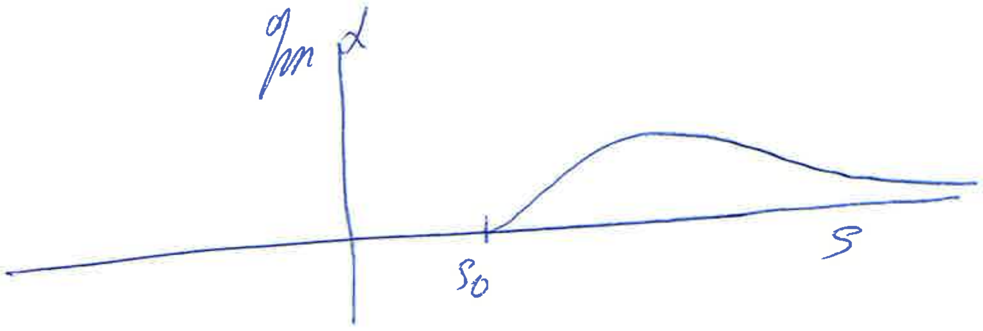
In this way Regge (1959) proved the validity of

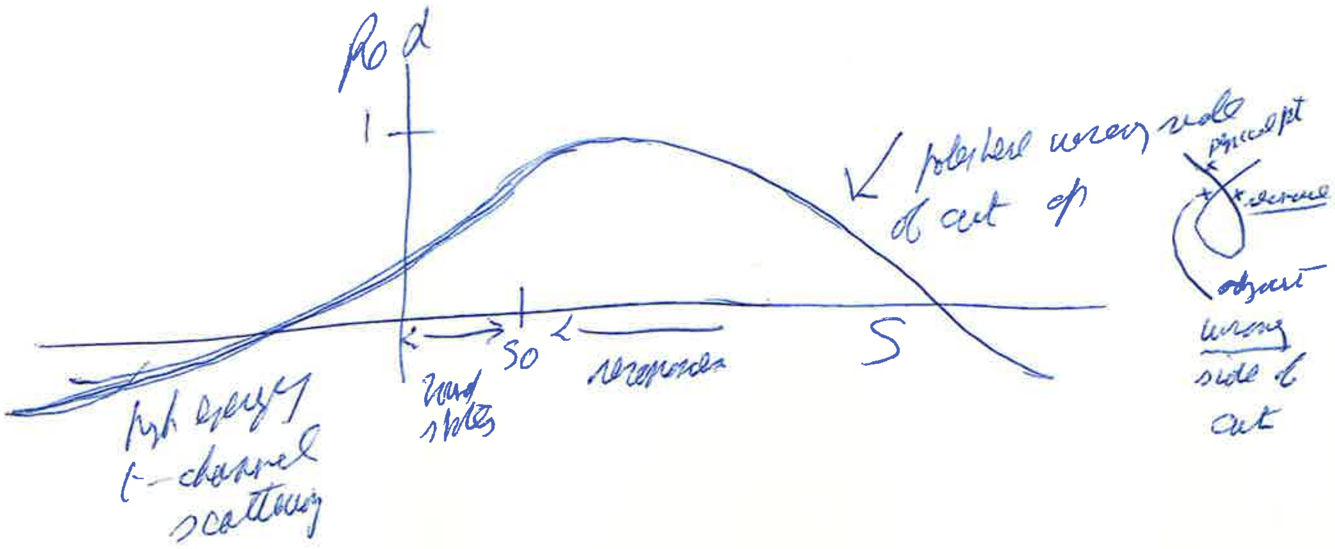
cf. Mandelstam representation in potential scattering

What do Regge-trajectories look like

$\operatorname{Re}\ell = \alpha(s)$







The Bootstrap Hypothesis may be formulated as:—
"The only set of particles (poles) consistent with the principles of maximal analyticity of the 1st & 2nd kinds & the actual set of strongly interacting particles found in Nature". Collins & Squires p. 140 (Regge Poles in Particle Physics).

Analyticity of 2nd kind may also be expressed in the form.

all poles are Regge Poles i.e. are moving poles in the J-plane, & all particles lie on Regge trajectories

N.B. When we continue $a(l,s)$ in l we continue the pole positions in l, thus

$$a(l,s) = \frac{\beta(l)}{s-\alpha(l)} \quad \text{—①}$$

~~Thus~~ where $\alpha(l)$ is an analytic function of l.

This is to be contrasted with

$$a(l,s) = \frac{\beta(s)}{l-\alpha(s)} \text{ — ② which defines}$$

the Regge poles.

If $\alpha(s)$ has an inverse function ② can be re-written in form ①

i.e. $l = \alpha(s)$ defines $s = \alpha^{-1}(l)$

this requires non-vanishing $\alpha'(s)$ in the neighbourhood where the inversion is to be made.

If $\alpha(s) =$ const. the inversion is not possible

If inversion is possible then pole can be regarded as occurring in s or J at will.

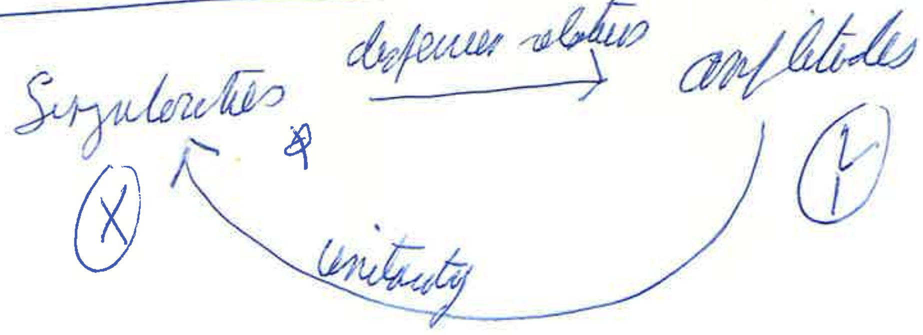
as example cf. A $f(z,w) = \dfrac{1}{z+w} \rightarrow \underline{\text{moving poles}}$

with B $f(z,w) = \dfrac{1}{z} + \dfrac{1}{w} = \dfrac{z+w}{zw} \rightarrow$ fixed poles

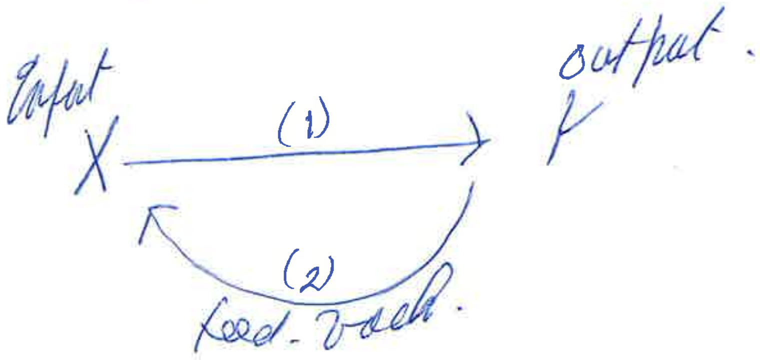
A is type of behaviour we have in s, J variables

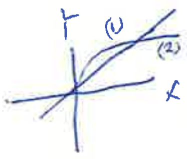
B . . . . s, t variables

Analogue of Bootstrap condition


Singularities
(X)
dispersion relations
amplitudes
(Y)
unitarity

cf with


Input
X
(1)
output
Y
(2)
feed-back



Ex for (1) might be. $Y = X$

(2) . . . $X = Y^2$

Soln is $Y = Y^2 \to 0$. $Y = 0$ or 1

so either $(X,Y) = (0,0)$ or $(1,1)$

Ambiguity here corresponds to C.D.D. ambiguity. To remove we might invoke on law of Nature. Soln must be invariant under reflection in the origin. This would eliminate the $(1,1)$ soln in favour of the $(0,0)$

this would correspond to introducing our second principle (maximal ambiguity of 2nd kind)

N/D Bootstrap is formulated in terms of partial wave amplitudes.

Using partial wave amplitudes $a_\ell(s)$

From singularity structure in s, t we write down dispersion relations for $a_\ell(s)$ (coupled eqns in general)

Schematically we proceed thus.

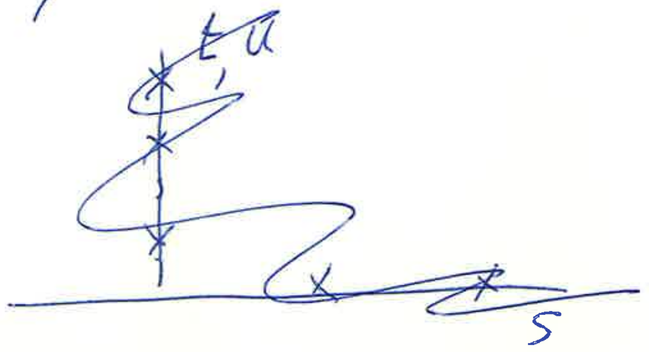
Take crossed-t, u channel particles (maybe resonances)

Assume no S-channel poles

now set of dispersion relation can be solved to yield particles (poles) in S-channel.

Substitute back in the singularity structure for $a_\ell(s)$ (cut on physical sheet, i.e. bound states) & re-solve for self-consistency

i.e. same poles come out of S-channel as are put into the crossed-channels.



For the N/D equations, putting the new singularities back into the equations does not alter the solution

So. different eqs have the same soln

Also C.D.D. ambiguity

different solns belong to the same Eq.

this ambiguity exists for $l = 0$ or 1 only, but must be removed if bootstrap is to work.

Chew, Frautschi remove ambiguity by using analyticity in angular momentum
$l = 0$ or 1 poles are continued along Regge trajectories from region $l > 1$, where ambiguity does not arise.

In general we write $\{\underset{\overset{\uparrow}{\text{output}}}{P_1, P_2 \cdots}\} = X_1 X_2 (\underset{\overset{\uparrow}{\text{input}}}{P_1 P_2 \cdots}) X_3 X_4 \cdots$

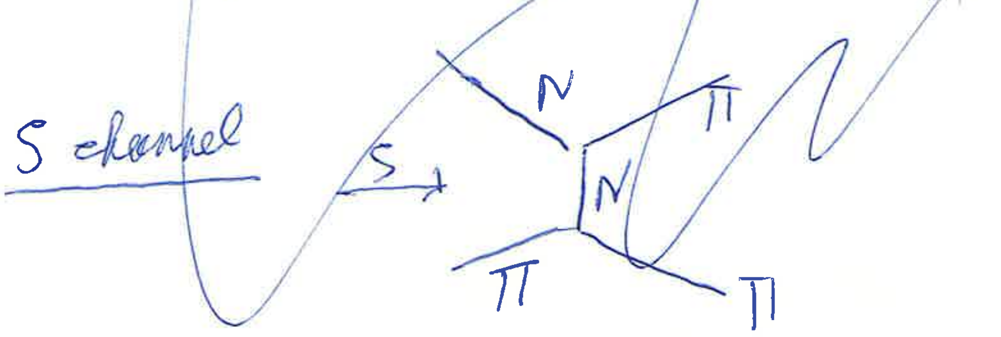
~~$P_1 P_2$. $Q_1 Q_2$ Q~~

~~$\{Q_1, Q_2 \cdots\} = X_1 X_2 (P_1,$~~

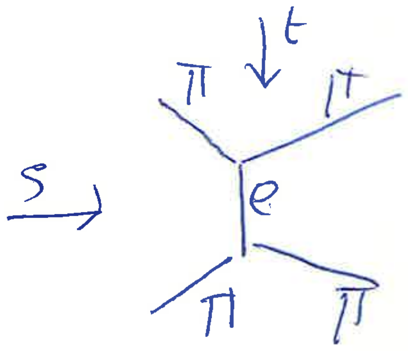
where scattering process is $X_1 + X_2 = X_3 + X_4 \cdots$
$P_1\, P_n$ are particles exchanged in crossed channels and also generated in the S-channel.

Ex ~~$\{N, \pi\} = \pi N\,(N, \pi)\,\pi N$~~

$\{N, \pi\} = N\pi\,(N\pi)\,N\pi$



(1) $\rho = \pi\pi(\rho)$



2 pions exchange a ρ-meson (in either t or u channel), which produces an interaction which leads to the ρ-meson pole reappearing in the S-channel.

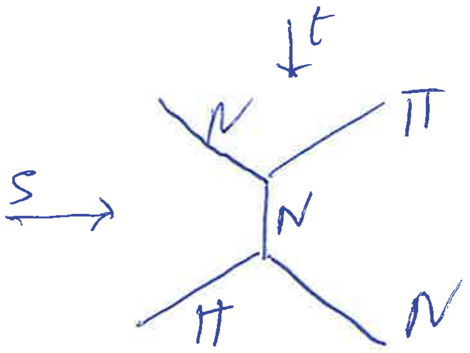
The ρ-meson is said to bootstrap itself

(2) In the first example the π was uninfluenced. Suppose universe consisted only of π's N

then consider the scheme.

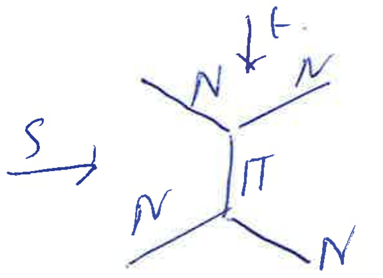
$$N = \pi N \ (N)$$
$$\pi = N\bar{N} \ (\pi)$$

The nucleon, pion individually bootstrap themselves and are also the interacting particles, thus



(N exchanged in t-channel only.

and.



(π exchanged in t or u channels)

fails in practice because

(1) exchange N does not give attractive force in N state for πN scattering

(2) π is tightly bound $N\bar{N}$ system involves exchange of ρ etc in addition to π to account for strong binding

(3) The reciprocal bootstrap

$$N = \pi N \, (N^*)$$
$$N^* = \pi N \, (N)$$

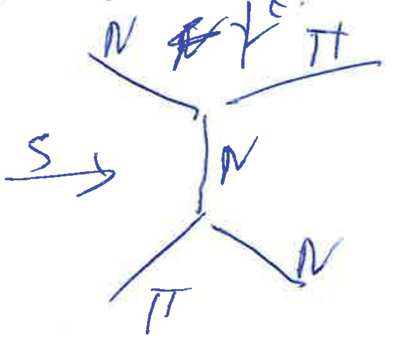
The N^* in the t-channel generates the N in the s-channel and N . . . t-channel . . . N^* . . . s-channel

we can more generally write

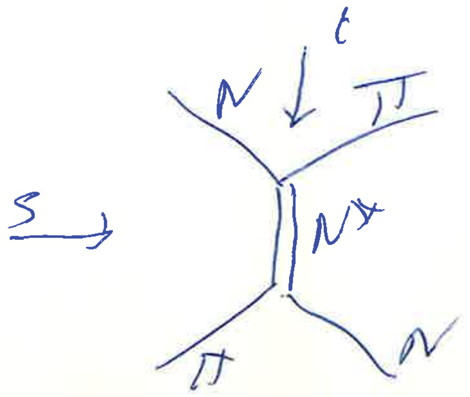
$$\{N, N^*\} = \pi N \, (N, N^*)$$

but the principal contributions are the reciprocal ones, not the self bootstrap of the previous example.

Hence



N^* exchange.



N comes out as bound state of N & π

But all these schemes are only partial bootstraps, since only a few particles and processes are included.

More generally every particle is produced by some process or reaction which it in turn helps to produce.

In particular notice how particles are part of themselves

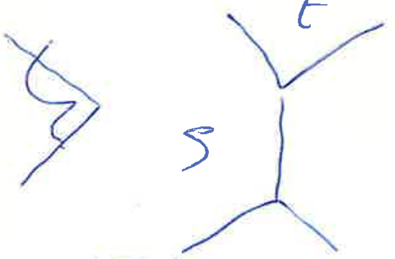
e.g. $N \rightarrow N + \pi$

the analysis of a nucleon actually leads to a nucleon and in a sense the part is equal to the whole.

The old philosophy of Anaxagoras where every atom contains part of every other atom is clearly worked out in the bootstrap philosophy.

Practical Bootstraps (after Collins & Squires Ch. VI)

scalar meson as in $\mathcal{L} = \lambda\phi^3$ cannot bootstrap itself (Collins 1965)



Ex of a theory for check. arbitrary particles cannot be calculated

for ρ bootstraps : Zachariasen (1961) found ρ self-consistent for $m_\rho = 350$ MeV, $\Gamma = 300$ MeV. True values $m_\rho = 750$ MeV, $\Gamma = 110$ MeV.

If m is given correct value couplings (relates to Γ) do not bootstrap.

N, N^* more successful. but results are pretty rough, factors of 2 or 3 out of expected values being common. (cf Gasio p. 430)

Elementary Particles & Greek Philosophy ㉔

Heisenberg: Physics & Philosophy, 1958 Ch. IV "Quantum theory & the roots of Atomic Science.

Milesian School: Thales introduced idea of a (i.e. Part of Ionia) fundamental substance (water) of which all other things are transient forms.

Anaximenes suggested air as primary substance.

Odd man out is Anaximander who denied the fundamental substance was any known substance — a more metaphysical notion, undifferentiated Being.

H. favours Anaximander point of view → all particles can be viewed some Universal substance, no one particle more fundamental than any other.

~~Ionian School~~ Heraclitus (of Ephesus) → all is changing (with invariance) & much nearer to modern views than the Atomists, Leucippus & Democritus who were influenced by the Parmenidean One.

Empedocles had 4 substances — variety due to mixture & separation of unchanging substances. (earth, air, fire & water)

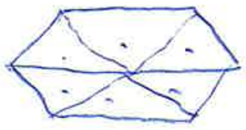
[cp Popper v. Kirk on Heraclitus]

→ for Heraclitus primal substance is fire

P.T.O.

Anaxagoras - assumes ∞ variety small seeds — all seeds are in everything — only proportions change — could lead to the Bootstrap

Plato in the Timaeus considers atoms, not as fundamental Beings, but as mathematical forms viz. equilateral, isosceles Δ's.
cf. SU3 & the building up of geometrical patterns
Δ's → hexagon



Redhead 1974

The Bootstrap and the Atomistic Programme

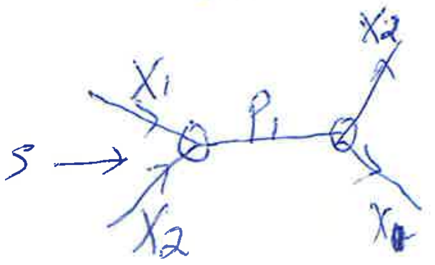
The Bootstrap can be described as featuring 3 roles for hadrons

1) constituent of a composite
2) "exchange" particle generating the force which binds the composite
3.) whole composite itself

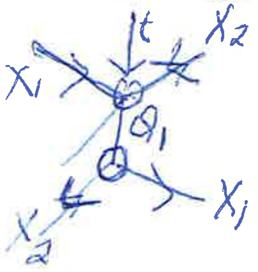
we could write e.g.

$$\{P_1\, P_2 \,--\} = X_1\, X_2 \cdots (Q_1, Q_2 \cdots)$$

	composites	constituents	"exchange" particles
identified as	pole in direct channel	reacting particles	poles in crossed channels

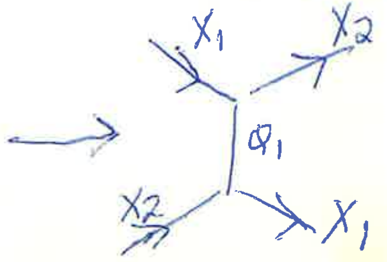


derived from



These are unitarity diagrams

or in terms of Feynman diagrams



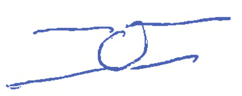
Exchange of Q_1 generates potential to produce bound state pole in s-channel

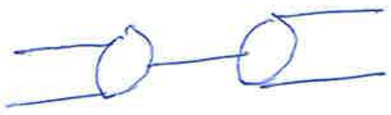
We now remark that the same poles P_1 may appear in many different vertices, so we can write

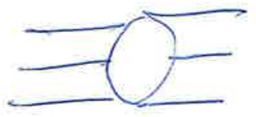
$$P_1 = X_1 + X_2 \quad \text{or} \quad X_1 + X_2 + X_3$$

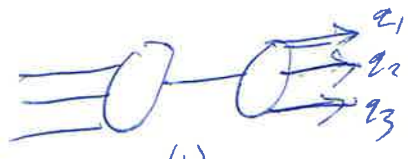
In general pole occurs when any scattering process can be regarded as made up of 2 successive scatterings "linked" by formation of a single particle.

These may occur in various ways.

e.g.  involves a pole which may be represented thus

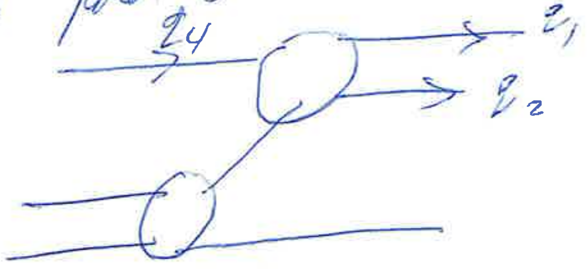


or  involves poles such as



(1)

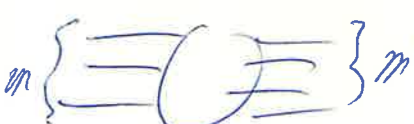
or



(2)

Notice in (1) pole occurs at $(q_1 + q_2 + q_3)^2 = M^2$

while in (2) pole occurs at $(q_1 + q_2 - q_4)^2 = M^2$.

For $n \to n'$ transition 

Lorentz ↑ mass ↑ shell

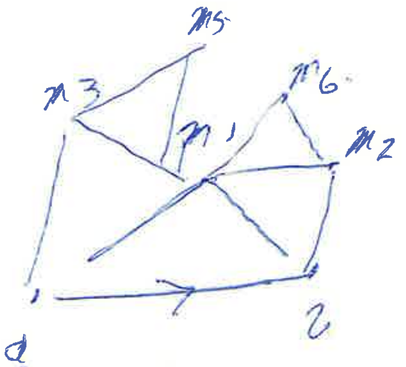
~~scatt~~ transition amplitude depends on $4(n+n') - 6 - (n+n)$ ↑ cons. energy, mom. -4

$= 3(n+n') - 10$ parameters

Ex $n = n' = 2$ there are 2 parameters (s, t e.g.)
$n = n' = 3$ . . . 8 parameters

poles may occur in any of the variables as illustrated for energy and cross-energy variable in above example.

S-matrix approach involves amplitudes for all states beginning in initial, ending in final state — network of interlocking amplitudes is rapidly generated.



and in general ~~Pole in~~ $\overset{\text{amplitude}}{}$ $a \to b$ involves all the other amplitudes as well.

The point about the bootstrap is that the sets $\{P\}$, $\{X\}$ and $\{Q\}$ are all
composite constituent force
the same set, the unique set of actually existing Hadrons.

Thus we write the bootstrap ~~symbol~~ schematically as

$$X_1 = \begin{cases} X_1 \\ X_a + X_b \\ X_a + X_b + X_c \end{cases}$$

$$X_2 = \begin{cases} X_2 \\ X_a + X_b \\ X_a + X_b + X_c \end{cases}$$

$$\vdots \qquad \vdots$$

This leads to some anomalies $A = B + C$
e.g. (1) ~~$C = A + B$ and B~~ and $B = A + C$
so A is part of B and B is part A
or (2) $A = A + C$, so A is part of itself

The meaning of an equation such as
$A = B + C$ is that if A is suitably "excited" it can fragment into B and C
but "containment" model only works if excitation energy required to separate constituents is ≪ rest energy of particles.

In fact, in view of possibility of creating & destroying particles characteristic of elementary particle physics it is best to think of energy contained in incident system

$$\underbrace{X + \underline{A}}_{\text{incident system}} = \underline{B + C} + X$$

is used to "create" the particles B & C
B and C may also be regarded as existing in "virtual" state i.e. short lived existence allows quantum fluctuation in energy required to produce the particles.
In this sense there is no anomaly in above examples when interpreted in terms of "creating" the

particles on the right, instead of "releasing" what is already present on the left of the above equations.

N.B.
$$\begin{array}{ccccccc} X & \Rightarrow & A & + & B & + & C \cdots \\ M & & m_1 & & m_2 & & m_3 \end{array}$$

is only possible if $M^{*} > m_1 + m_2 + m_3 \cdots$

But $X + A \rightarrow B + C \cdots + X$ can in general satisfy conservation of energy & momentum for suitable momentum of X.

cf Chew (1968)

"Nature is as it is because this is the only possible Nature consistent with itself!"

The bootstrap hypothesis can be related to
a) Leibniz, b) Anaxagoras.

Leibniz Principle of sufficient reason (Monadology para 32)
says "no fact can be real or existing and no proposition can be true unless there is a sufficient reason, why it should be thus and not otherwise, even though in most cases these reasons cannot be known".

i.e. there must be a reason for everything

this really means no arbitrary parameters in scheme of Nature. (& all true statements are analytic their falsity involves contradiction)

2 versions of views of Leibniz 1) many possible non-contradictory, i.e. self-consistent worlds, which one exists is due to God's choosing the best possible world, 2) even existence is rationally determined, those objects exist which are compatible with the greatest number of other objects (cf Russell's analysis)

But if monads are identified with elementary particles, they do not interact — each monad determines analytically all its own properties, but each mirrors the universe, pre-established harmony among the monads

Anaxagoras

Essentially theory can be given in two forms

1) Every substance infinitely divisible into elementary seeds i.e. molecules. Each seeds contains portions of all other seeds. portions are mathematical divisions of the seeds, which correlated to physical division. (cf. Zeno's paradox of infinite continuity) portions are like atoms in molecule. cp Kirk, Raven

2) Better interpretation favoured by Gershenson, Greenberg (theoretical physicist + classicist) is each substance is uniform mixture of infinitesimal grains or seeds (or portions) of all different substances. there is no aggregation into ~~seeds~~ molecular "seeds", but however small a sample we take all "seeds" will be present.

Taking "molecular" form of the theory we the formulation every seed contains a portion of every other seed.

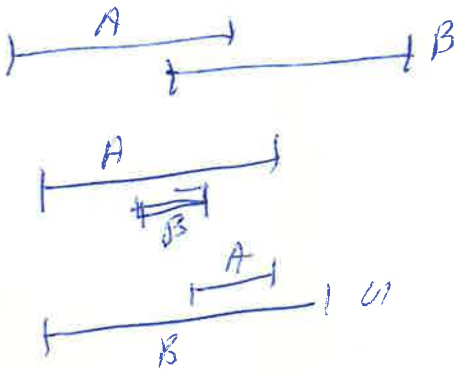
* cf with 2 sets A & B may each include
subsets of the other
but they cannot each include
the whole ~~of~~ other set

Yes

But if

then

is impossible



This is a sort of extreme bootstrap.

N.B. Major difference is

$$X = A + B + C$$ –

is only allowed if Quantum numbers are conserved

e.g. $\overset{(-1)}{\Lambda^0} + \overset{(0)}{\bar{N}}$ cannot produce a pion since strangeness not conserved although baryon no. is.

Every particle is linked to every other particle but only in combinations which preserve Q. No's.

eg $\pi = \Lambda^0 + \bar{\Lambda}^0 + N + \bar{N}$ is possible

so Λ^0 and $\bar{N}$ are "involved" in π, but not in the simple form $\pi = \Lambda^0 + \bar{N}$.

Notice Anaxagoras is essentially a containment model but contradictions are avoided by formulation seed contain portions of all seeds. *
in place of seeds contain all seeds.
which would be closer to the bootstrap formulation ~~into~~ expressed in terms of "creation" in place of "containment". The Concept of creation is

definitely in contradiction to Anaxagoras, who wanted to explain change in terms of unchanging substances, following ideas of Parmenides on contradictory nature of change.

In this respect the bootstrap, indeed elementary particle physics generally is much closer to Greek philosophers in which substances can change their forms (Milesians) as e.g. Thales, Anaximenes, Anaximander and particularly Heraclitus

Anaxagoras is much closer to Empedocles and Leucippus, Democritus all of whom derive from the Parmenidean reaction to Heraclitus.

Alternatives to the bootstrap Varieties of Fundamentalism

1.) some particles are elementary particles, or could be nucleon or pion ($J \leq 1$)

all particles can now be derived from arbitrary properties of the aristocrats but the relationship of "composite" particles to the aristocrats is via the cycle of Bootstrap-type equations — quite different from simplistic containment model derived from classical atomistic programme.

2.) Constituent models for the hadrons:

this regards the hadrons as composed of other simpler particles, fields, quarks or partons

thus we write $X_1 = Q_1 + Q_2 + Q_3$

$X_2 = Q_1 + q_2 + q_3$

$\vdots$

and this set of structures is a "solution" of

Imp

* at any rate for some processes
but cf hadronic decay such as $N^{*} \rightarrow N + \pi$
$N \rightarrow N\gamma + \pi$
in which $\pi\bar{\pi}$ pairs are created.

the bootstrap eqs. $X_1 = \begin{cases} X_1 \\ X_a + X_2 \\ \vdots \end{cases}$ $X_2 = \begin{cases} X_2 \\ X_a + X_2 \\ -\,- \end{cases}$

i.e. the hadrons are "explained" in terms of quarks at a "deeper" level of structure.

For this to make sense we must suppose either that quarks are elementary particles as in 1) or that the eqns

$X_1 = q_1 + q_2 + q_3$ etc are to be understood in the [illegible] "containment" sense*, which would mean, e.g. that at energies available quarks are not capable of being created (or virtual processes quarks we are to assume that their creation leads to very short range interactions of importance at very short wavelengths (high energies) capable of ~~to~~ "seeing" the structure in the interaction)

Hadronic vertices interpreted in terms of re-arrangement of "unchanging" quarks (some features at any rate (cf e.m. interactions)

Constituent models lead us very deep
into infinite regress

Quote from Kogut & Susskind (1973)

" It seems to ~~[illegible]~~ very unlikely that partons are really point particles"

" A parton will be resolved even further into yet another hierarchy of constituents . . . "

i.e. at shorter & shorter distances we may uncover more & more structure.

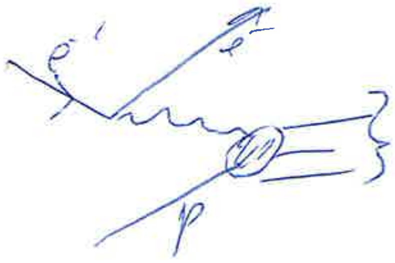
but note that $q\bar{q}$ pairs must be created to account for $N^* \to N + \pi$ e.g.

i.e. baryons are qqq structures & mesons are $q\bar{q}$ structures

Two main indications for this model.

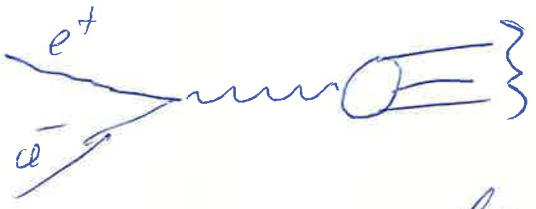
1) Missing triplet representation of SU(3) octets & decuplets identified. What about the triplets? → Quark model.
qqq & $q\bar{q}$ and l-excited states explain the masses.

2) Phenomenological analysis of deep inelastic scattering of electrons by protons reveal point-like structure (partons) which may be identified with quarks.



partons held together in proton by gluons

Many good predictions but fails in process



also some discrepancies in scaling laws

$d\sigma$ function of (ν/q^2) only.
H DE/EE only

Note Quarks could agree with bootstrap — come out of self-consistence scheme.

3) Both 1) & 2) regard fundamental object as another form of matter. In this respect they are a Thalean form of fundamentalism

We turn now to

3) Anaximandrian form of fundamentalism.

Fundamental object is something different from ordinary matter, undifferentiated Being, or as we might say energy, of which the particles are manifestations:

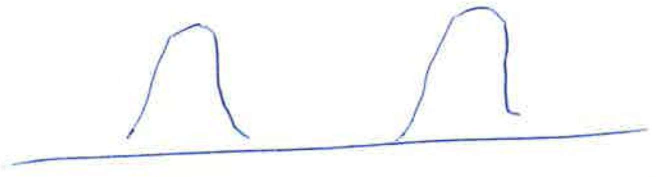
. This leads to

Unified Field Theory approach (cf Heisenberg)

Schematically we write

$$X_1 = X_1(F)$$
$$X_2 = X_2(F)$$
$$\vdots \qquad \vdots$$

* singularity picture could correspond to states of definite position for the particles, not eigenstates of energy which look like travelling waves. Such localized states have very large energy density in small region of space.



superposition of 2 1-particle states is no longer a soln, but interaction → "scattering" 2-particle states.

i.e. all particles are explained in terms of a single principle, the unified field F.

F satisfies a non-linear eq. of motion.

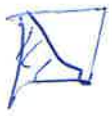
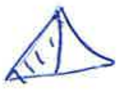
Particles represent singularities of different sorts in the field. (or eigenstates?) *

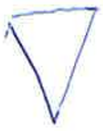
quantisation of the field leads to ~~quantizing the amplitudes of these singularities i.e.~~ to a theory of definite nos of particles of definite sorts.

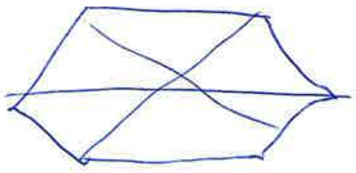
Rothstein gives the analogy of particles as knots on a string

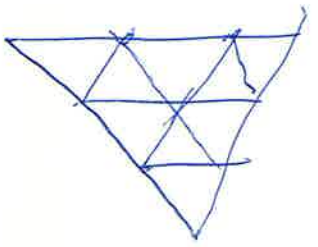
the knots are the natural particles we can "see" but their true nature can only be apprehended by studying the reality behind the appearance viz the ~~string~~ [illegible] of the string from which the knots are made.

4/ Mathematical Atomism

cf Plato's Timaeus regular solids
cube, octahedron, icosahedron, dodecahedron,
all constructed from 2 sorts of △
viz half a square and ½ + equilateral △
thus  and 

this parallels the *eight fold-way* of
Gell-Mann + Ne'eman SU(3)
where patterns are built of simple patterns
thus *triplet* 

octet 

decuplet 

Chew: (1970) "I would find it a crushing disappointment if in 1980 all of hadron physics could be explained in terms of a few arbitrary entities — we should then be in essentially the same posture as in 1930

. . . to have learned so little in half a century would to me be the ultimate frustration."

Short comings of the Bootstrap

1) It makes life very difficult for physicists.

2) Partial bootstraps may be infeasible to study or at the extent they may be spoiled by including new particles. (Partial bootstraps may also involve a restricted form of causality (say elastic)) (—this can make a partial bootstrap possible)

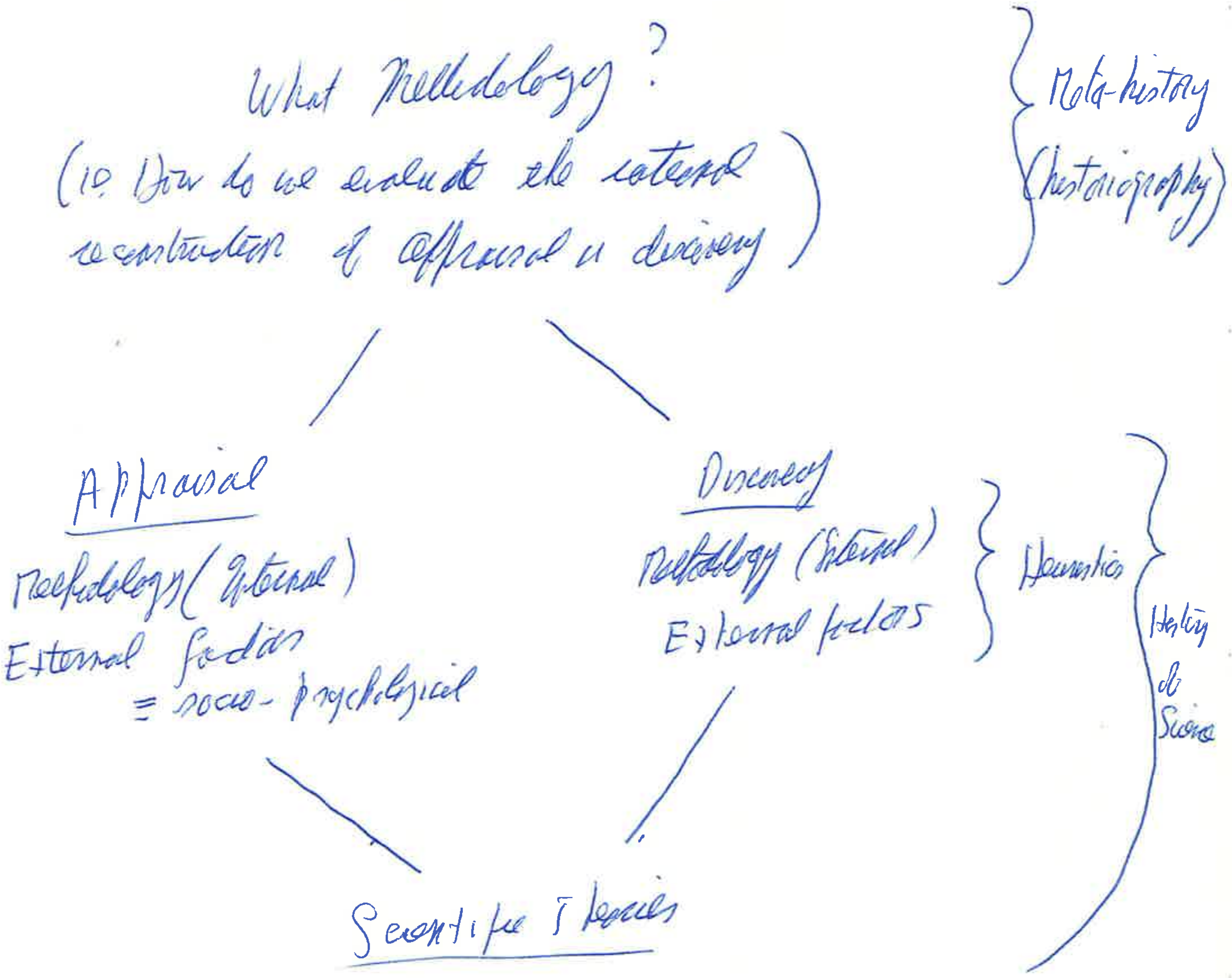
3) Also how can idea be tested? Is it in fact unscientific?
a) potential proof of consistency
b) all particles lie on R. trajectories (analyticity in [illegible])

4) What about bootstrapping photon, electron — Chew claims succeeded with resonant process — but what about the [illegible] of the neutrino?

5) Chew claims complete bootstrap would demand for self-consistency confronting the elusive concept of observation and possibly even of consciousness (cf Wigner)

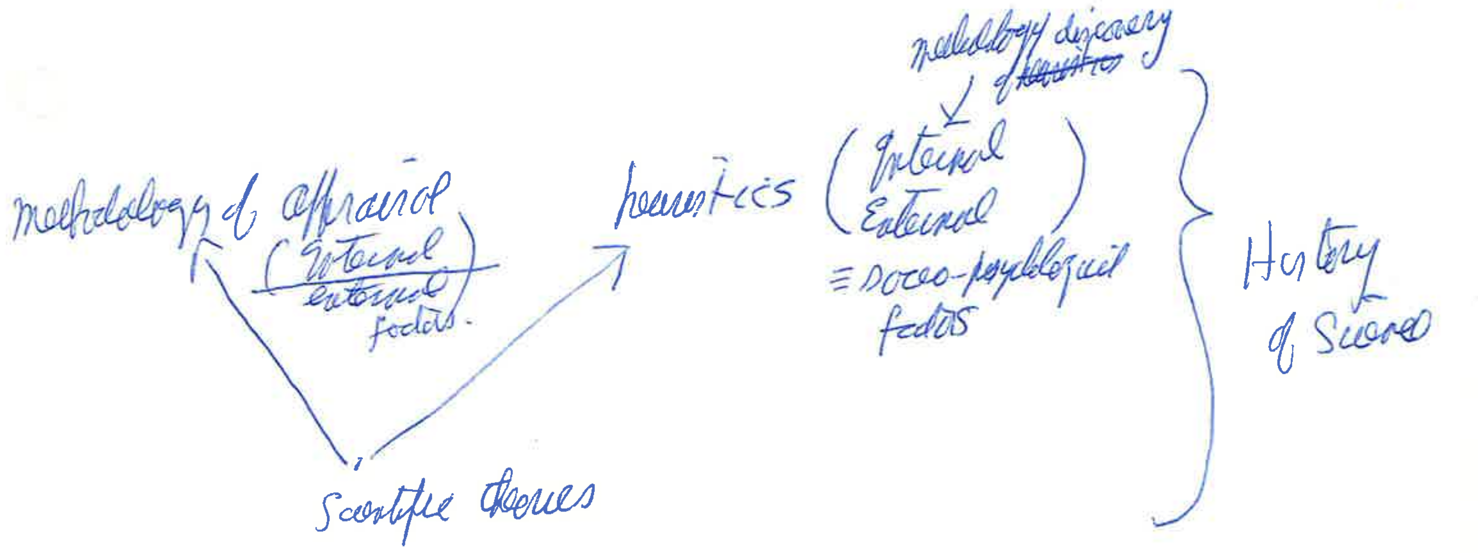
" Quote from Chew " I would find it [illegible] the ultimate particles".

History & Meta-history


What Methodology?
(ie. How do we evaluate the rational reconstruction of appraisal & discovery)
Meta-history (historiography)
Appraisal
Methodology (Internal)
External factors
= socio-psychological
Discovery
Methodology (Internal)
External factors
Heuristics
History of Science
Scientific Theories

The place of heuristics in the analysis of scientific progress

How do we evaluate the rational reconstruction of appraisal or discovery? } meta-history (historiography)


methodology discovery of ~~heuristics~~
methodology of appraisal
(internal
external factors.)
heuristics
(Internal
External)
≡ socio-psychological factors
History of Science
scientific theories

Two ways of doing meta-history

1.) argue for the correctness of a particular methodology of appraisal or of heuristics

2.) apply Falsifiability : (a) naive
(b) sophisticated

Methodologies of appraisal and of heuristics (together labelled rationality theories) are to be rejected if they are inconsistent

with the accepted 'basic value judgment' of the scientific elite' (Lakatos 1971)

But Lakatos denies that there is such a thing as a methodology of heuristics, only for appraisal.

This point of view is contested in Post (1971) [Correspondence, invariance, heuristics]

p. 250: "It is hoped that this paper will stimulate philosophers of science to study the heuristics of theory construction in particular cases".

In Lakatos's methodology of historiographical research programmes the possibility is envisaged of research programmes in rationality theories (as applied to appraisal) – may "predict" novel historical facts – i.e. rationality where previously none seen – in this case the scientific elite may have to change their minds

cf. [If theory clashes with experiment, so much the worse for the experiment]

The Science of Heuristics

By *heuristics* is meant the study of the process by which discoveries are made, i.e. part of the way in which scientific progress is achieved. Other part is *methodology of appraisal*. Essentially heuristics belongs to the *history of science*.

To study a *science of heuristics* involves trying to do two things :- (1) ~~cond~~ producing a *rational reconstruction* of ~~progress~~ discovery in science (2) apply some rational falsifiability criterion for testing such a rational reconstruction. This is supplied by Lakatos' meta criterion — acceptability by the scientific élite, but Lakatos applies not naive methodological falsification but the sophisticated variety, wherein a rationality ~~theory~~ ~~programme~~ programme is not rejected until it is supplanted by a more progressive one. *Also* one must give the scientific "élite" time to consider their verdict on a putative rational reconstruction

Classification of methodologies ~~theories~~ of ~~heuristics~~ appraisal

RATIONAL	IRRATIONAL
(1) Inductivism (i.e. Justificationism) acceptable → new theories are empirical generalizations from the facts of Newton & Ampère	Kuhn and the paradigm shifts Appraisals & Heuristics external i.e. appraisal & heuristics coincide
(2) Conventionalism acceptable → new theories are more economical (simpler) than old theories cp Ptolemy → Copernicus	again appraisal & heuristics coincide
(3) Falsificationism Importance of crucial experiments as falsifiers of old theories cp Agassi "Towards an historiography of Science"	theories appraised in respect of falsifiability and successful novel predictive power quite distinct from heuristics

(4) Research Programmism

Progressive research programme replaces a degenerating one. But heuristics of how ~~the~~ programmes modify and change into another programme is relegated to empirical psychology & sociology of discovery.

Our Science of heuristics combines features from several people

A From Kuhn it accepts paradigm shifts but only on the grand scale (as such shift can be identified with a reformulation (directional shift) but is then contrary to meta criterion as a rational reconstruction): So far as approach is concerned paradigm shifts may not be irrational but explicable in terms of research programmes

B From Lakatos it accepts hard core's protective belt

C From Popper it rejects normal science and also the working of the positive heuristic in a research programme, as essentially 2nd rate features of science

D This leads to Toulmin's concept of an unit of variation in an evolutionary approach to the growth of knowledge

E This unit of variation is identified with a micro research programme.

Our main concern is to ask the question.
How are research programmes (micro or not) related? Can we find any rational connection between rival research programmes? i.e. How do r.p's correspond – this now leads back to Post's principle of Correspondence which authorises the conservative element in new theories, which borrow structure from old theories.

This feature of correspondence is now analyzed under the general term directional shift, produced either by a polarizing phenomenon or by a polarizing property of some model of the old theory

The Analytic S-matrix and the heuristics of directional shifts within a paradigm

We have already discussed the development of relativistic quantum field theory as an instance of a directional shift within the paradigm of Q. Mechanics (Fock-space) followed by a stretching of the theory in the direction of the following phenomena (variable number of particles)

The importance of the computation gap in separating the hard core protective belt from practical prediction has also been stressed.

The case of the Feynman diagrams is seen as a directional shift (reformulation) of field theory which did not involve a stretching of the theory, but which served essentially to close up (to some extent at any rate) the

Computation gap.

We may at this point distinguish two senses of the word 'Model'.

(1) In terms of our previous diagram. different types of approximations, attributes at point 7 in the Computation gap, produce different approximate theories which refer to under the generic term Model$_1$

(2) We may introduce some very simple structure in the protective belt of overriding hypotheses which enables exact (or relatively exact) predictions, i.e. close up the computation gap. The resulting approximate theory (i.e. approximate if there is still a small computation gap) is referred to as a Model$_2$, & the full theory which has more realistic underlying hypotheses (e.g. we might choose

some very simple type of field equation that is exactly soluble.

Essentially Model$_2$ is a special case of Model$_1$, since, in general we can regard the simplified auxiliary hypotheses (changes at points X in the protective belt) as the first step of approximating the complete theory — effectively ~~so~~ changes in X to produce a Model$_2$ can be regarded as ~~chang~~ moved into the computation gap so as to be subsumed under Model$_1$ treatment.

But in practice the distinction between a Model$_1$ and a Model$_2$ is usually fairly clear.

$\times$ Eg

$$f(x) = \sum_{1}^{\infty} a_n(x) \simeq \sum_{1}^{N} a_n(x)$$

$$\underset{x \to \phi}{\mathrm{Lt}}\ f(x) \not\simeq \sum_{1}^{N} a_n(\phi)$$

↑

not a good approximation

We can now suppose that as a result of approximating the supposed complete theory (either at X or Y) we can arrive at some conclusion P, which we suppose, ~~for experimental~~ ~~to~~ enables us to give a clear phenomenological account of some group of phenomena. C say.

Now P is not a conclusion of the original theory T, but only of the approximate theory* T_1. where T_1 is equivalent to $T \wedge A$

we can write this as $$\underset{\substack{\uparrow \\ \text{approximate} \\ \text{theory.}}}{T_1} = \underset{\substack{\uparrow \\ \text{theory}}}{T} \wedge \underset{\uparrow \text{ approximation}}{A}$$

We can however proceed to introduce P as an axiom ~~in~~ for a modified $T \rightarrow T'(P)$ which embodies P as an essential part of its structure. These can be described

by saying that P has been made model independent by fiat ie. it has now been incorporated in the hard core, so that any subsequent models ie must to incorporate P ie ~~to~~ be models for T'(P)

It is possible that T'(P) is ~~identical~~ ~~with~~ equivalent to T in a new formulation. In such a case we have merely achieved a reformulation, ie. a dialectical shift, but the usual circumstance is that we cannot for reasons of mathematical difficulty prove T = T'(P) and so adopting T'(P) may involve a stretching of the theory in the direction of the polarization property P which here replaces the concept of a polarizing phenomenon, which was previously

used to explore the heuristics of directional shifts and stretches.

The whole process can be summarized as follows in a schematic form.

$$\begin{tikzcd}
\overset{\text{original theory}}{T} + \overset{\text{approximation product of model 1,2}}{A} \arrow[r] \arrow[dd] & \overset{\text{approximate theory i.e. model}}{T_1} \arrow[r] \arrow[d, dashed] & \overset{\text{property of the model}}{P} \text{ (explains } C\text{)} \\
& \underset{\text{mathematical extension of } T}{T_1'} \arrow[r, dashed] & P \\
\underset{\text{new theory with } P \text{ incorporated as axiom.}}{T'(P)} & &
\end{tikzcd}$$

Ex 5 (i) In the analytic S-matrix

T is rel. Q. field theory
A is Feynman diagram perturbation approach.
T_1 is a class of Feynman diagrams
P is analytic properties of Feynman diagrams
C is dispersion relations connecting matrix elements of the S-~~matrix~~ operator. (Re, Im parts forward amplitudes)
T'(P) is the axiomatic analytic S-matrix theory

② Current Algebra (Gell-Mann 1962)

T is field theory

A is a special model for the fields (essentially equivalent to the quark model)

P is commutation properties of hadronic currents coupled through the weak interactions

T'(P) is current algebra formulation in which the particular current commutators are elevated into axioms

C in this case involves understanding of the connection between vector and axial vector coupling constants in β-decay via the Adler Weisberger Sum Rule. (1965)

In the case of the analytic S-matrix we have a very clear example of another new feature in the Heuristic Programme. The Property P is essentially a mathematical property (analyticity) of an extension of T_1 i.e. it is a property of a structure T_1' in which T_1 is embedded in the mathematical sense.

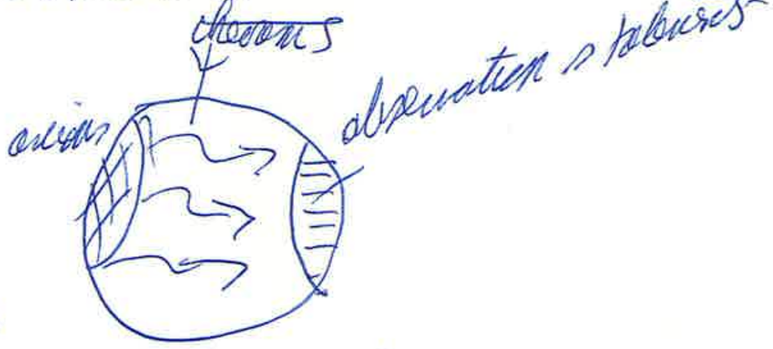
$T'(P)$ is now a theory in which the axiom P defines a physically 'real' structure in terms of properties of a mathematical extension of such a structure. This introduces concepts which go essentially beyond the class of physical concepts which can be identified with a theoretical ontology in Watkins's sense.

We now proceed to analyze the new feature of formulating theories which depart essentially from an approach based on the direct axiomatization of physically 'real' concepts.

Relation of Mathematics to Physics

We start from a physical theory T with axioms theoretical terms & observation terms.

Typically thus:–



Realist view

all terms have reference.

cf positivist approach (= all terms

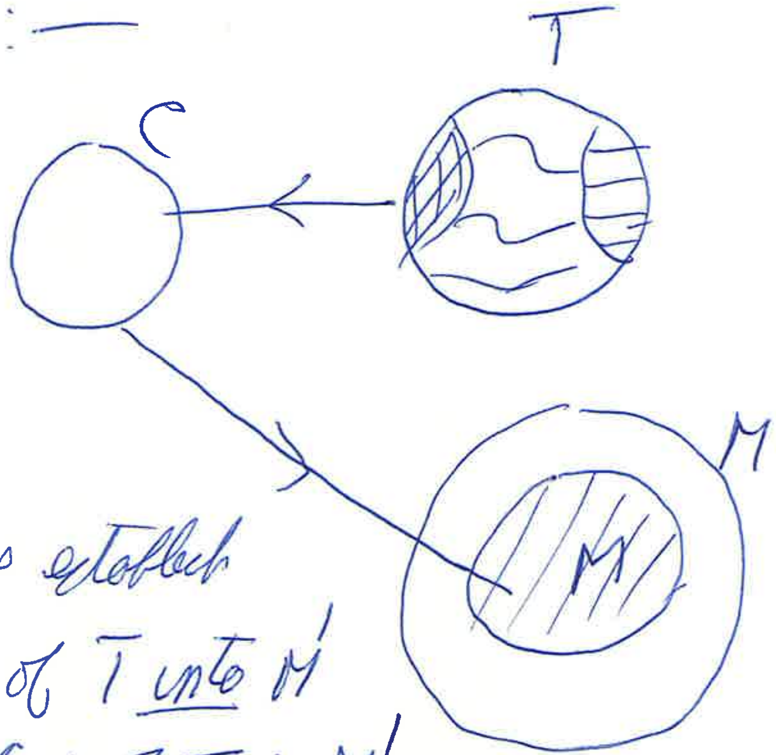
= all terms are observation terms

We abstract the uninterpreted calculus C of which T is a model.

Since T is supposed a complete description of the model, C will be a categorical formal deductive system (possibly we may want to formalize the theory to express it ~~say~~ in a 1st order formalized ~~predicate~~ language, but this may lead to the usual difficulties ~~arise~~ arising from the lack of expressive power of formalized languages → non-standard models & incompleteness.)

Thus we suppose C to be studied as an informal axiomatic system (i.e. unformalized yet, not formal)

Now look for a model of C in a mathematical system M, which may itself be embedded in a larger system M'. Thus:—



In this way we establish an isomorphism of T into M' i.e. an embedding of T in M'

M' is an extension of M supposed conservative i.e. no new theorems but not simple, i.e. no new symbols.

Two approaches to Mathematical Physics

① It is accidental ~~that~~ part of the structure of M' models the structure T

Physical theories are basic and should be axiomatized directly — they have nothing to do with mathematics

② It is characteristic of Science that theories T always model mathematical systems M. Mathematical theories are basic & express essential features of reality

But why are some mathematical theories "correct" others false. Platonist view proves too much — Conventionalism allows too little (simplest is only conventional, which is difficult to explicate).

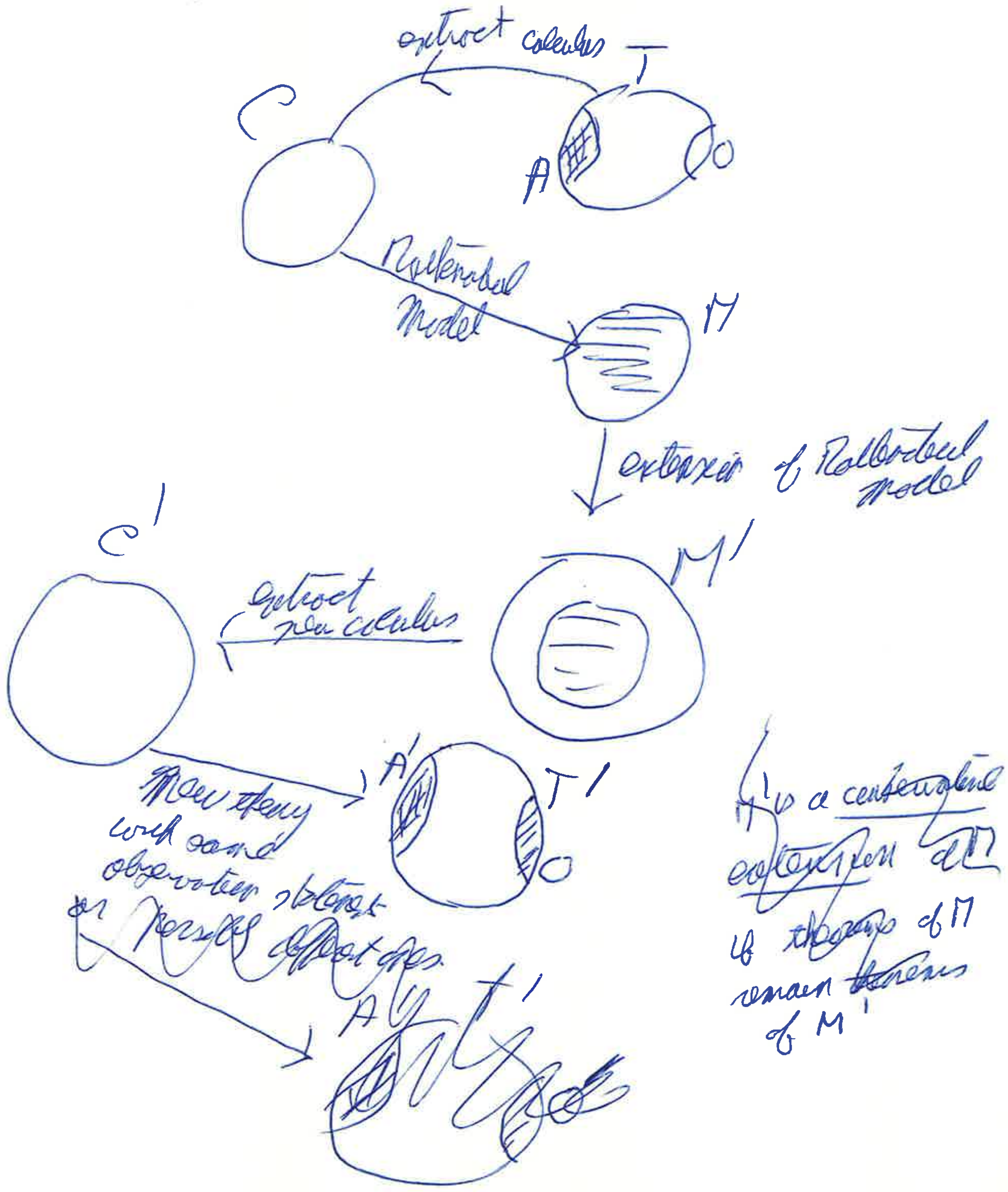
However Einst. etc believe mathematical considerations are very important — good example is

analyticity which is essentially a mathematical requirement (cf also General relativity).

Notice how by going from M to M' i.e. from ~~real valued functions to~~ ~~complex~~ the real variable to the complex variable, we can state properties or assumptions about functions on R very succinctly in terms of the theory of analytic functions of a complex variable. So mathematically there is much to be gained by considering M or M' although from the point of view M' involves 'unnecessary' "surplus" structure.

If we think that theories are best described as partially interpreted ~~calc~~ deductive systems

then we can extend M' back into a physical theory in the following manner


extract calculus
T
C
A
O
Rationalized model
M
extension of Rationalized model
C'
M'
extract new calculus
A'
T'
O
New theory with same observation statement or possibly different ones
A'
T'

M' is a conservative extension of M
the theorems of M
remain ~~theorems~~
of M'

By modifying M' we may however suggest modification ~~of new axioms~~ in the structure M which may now lead to ~~an M'~~ T' and new observation statements O'

Hence another way of analysing the reformulation (a directional shift) ~~is~~ of a theory under a paradigm followed by its stretching is by introducing a term altering the surplus structure of some extension of a underdetermined model of the old theory.

This is the situation which obtains in the introduction of the analytic S-matrix.

V. ~~Heraclitus~~ – [illegible] [illegible] is [illegible] –

In summary, then, the history of theoretical development of elementary particle physics during the last fifty years ① has allowed us to explore a variety of ways in which the heuristics of scientific progress can be understood, and ② has enabled us to present a number of points of view concerning the ultimate structure of matters.

Essentially we have learned that the success of a paradigm or research programme is largely dependent on its flexibility and the possibility of adapting it in accordance with its positive heuristic so as to provide an increasingly adequate language or theoretical frame within which to understand ~~the~~ or describe the experimental results, which continually challenge the programme.

but paradoxically too great a flexibility leads to a sense of ad hocness which militates against the aesthetic appeal of the programme.

These two opposing features

flexibility → possibility of survival

rigidity → aesthetic appeal.

have to be carefully balanced in the ideal research programme, which should be seen as a successive series of approximations to a natural order of classification of phenomena in Duhem's ~~terms~~ wording or to an increasing degree of verisimilitude in ~~Popper's terms~~. Popperian terminology.

With regard to the ultimate structure of matter, the various points of view that can be adopted, reflect basic metaphysical positions which have been essentially debated throughout the history of philosophy, and the conclusion we are driven to is a Kantian insistence on ~~essential limitation~~ the limited variety of conceptual frameworks that is apparently open to the human mind to hold and to investigate.

Quotation from Chew: The Analytic S-Matrix.
p. 2.

.. In a deep sense, physics is based on analytic functions. It is pointless to seek a logical origin for this circumstance. Physical theory cannot be based on logic; it is always a matter of guesswork based on observation of nature. One cannot, for example, argue that it is logical for classical mechanics to be expressible through second order differential equations. this simply is the scheme that works.

Feynman R.M.P 20 367 (1948)

Quotation Title "Space-Time approach to Non-relativistic Q. Mechanics".

"The formulation is mathematically equivalent to the more usual formulations. There are, therefore, no fundamentally new results. However, there is a pleasure in recognizing old things from a new point of view. Also there are problems for which the new point of view offers distinct advantages. . . . In addition there is always the hope that the new point of view will inspire an idea for the modification of present theories, a modification necessary to encompass present experiments".

Quotation of Bernstein: Elementary Particles and their currents on correspondence or in Philosophical Lecture

It is probably no exaggeration to say that all of theoretical physics proceeds by analogy. In constructing new theories one takes over those features of the preceding theories ~~that~~ that appear "universal" and, in analogy, one constructs the new theories to look as much as possible like the ~~old ones~~ preceding ones. There are two very good reasons for doing this. In the first place the human imagination, is, after all, rather limited. The prior theories provide the material on which the imagination feeds. Thus even the most 'radical' new theory retains some of the flavour of the ~~present one.~~ preceding ones. In the second place the older, accepted theories, must have had a domain of validity, and one of the conditions put on any new theory is that it agrees with the former chosen their common domain of validity — the "correspondence principle".

Quotation from March : Quantum Mechanics of Particles & Wave Fields - 1951)

Since its inception nearly a quarter of a century ago, quantum Mechanics has been the subject of many books. Notwithstanding this abundance of books, if someone, not a theoretical physicist & not thoroughly acquainted with Modern Methods of analysis were to attempt to digest a current article dealing with Nuclear forces or cosmic rays, relying for help on the available books on the subject, he would discover to his dismay that modern q. mechanics differs radically from that which he finds in the text books. Even the terminology is different. What, e.g. is the p.s. field which in some obscure way seems to be intended associated with a particle? He will not find the answer in the information at his disposal. More confusing to him is the electrification of a field in the new mechanics. It seems evident that what is meant is a real field possessing energy & momentum. Yet the text books attach a purely symbolic meaning to the wave field of a particle, picturing the field concept ~~as~~ merely as a probability function

Goudsmit P.R. 15th July 1973 D 8 p 351.

Important (repeated following No of P.R.)

Announcement

Acceptable papers must meet these requirements:

1.) Explicit assumptions stated in mathematical form

2.) these must lead to the explanation of hitherto unexplained observations

or 3.) expose new relations between known data or theories

or 4.) simpler, fewer assumptions are made than in existing theories

5.) The new assumptions must be shown not to contradict existing experimental facts

6.) Possible new consequences of the assumptions must be demonstrated & possible experimental tests described.

" Physics is an experimental science. In spite of the ~~present~~ temporary hegemony of theorists, no physical theory is significant unless it can be related to experimental data."